# Transformations

## Teacher Notes & Answers

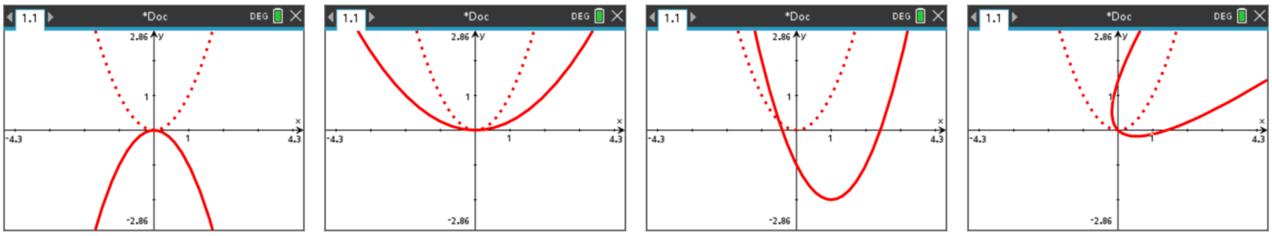
7 8 9 **10** **11** **12**

TI-Nspire™ | Investigation | Student | 50 min

## Introduction

Transformation means a change in form or appearance. Common transformations when dealing with functions include:

**Reflection** **Dilation** **Translation** **Rotation**



The aim of this activity is to provide an understanding of the algebra underpinning transformations. The technique involves the consideration of a single point and the effect it has on the general form or appearance of an entire family of points defined by a rule or function. A video tutorial is available to help set up your TI-Nspire file.

https://bit.ly/TI-transformations

## Set up

Open your “Transformations” document created using the video link above.

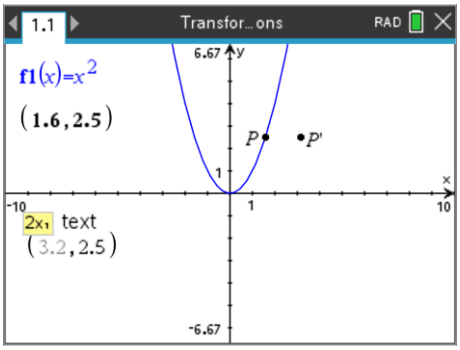
Point P $(x_1, y_1)$ is on the parabola: $f_1(x) = x^2$

Point P has undergone a transformation such that:

$P'(x', y')$ such that: $x' = 2x_1$ and $y' = y_1$

The text tip on P′ provides the transformation details.

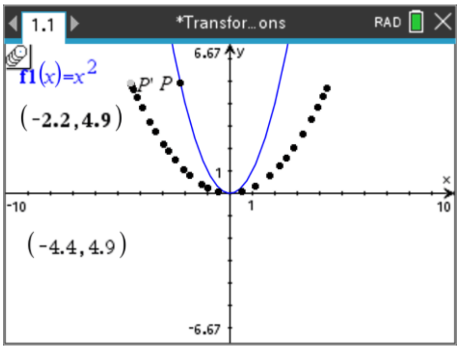
Edit the transformation for your point P′ to match these conditions.




Drag point P along the parabola and observe the coordinates of P′.

Point P′ is described as a dilation, “parallel to the x axis” or “away from the y axis” by a factor of 2.

In the screen opposite, the path of point P′ has been traced using the Trace (Geometry) tool.






Author: P. Fox

## Determining Equations

### Question 1.

a) Given $x' = 2x$ , $y' = y$ and $y = x^2$ , determine the relationship between $x'$ and $y'$ . Check your answer using your calculator and the corresponding transformation tools on the calculator.

Answer: $y' = \frac{(x')^2}{4}$ or $y = \frac{x^2}{4}$

b) Based on your answer to the previous question, describe the transformation from $y = x^2$ to $y = 4x^2$ . Test your answer using your calculator and the transformations file.

Answer: Dilation parallel to the x axis (away from the y axis) by a factor of ½.

### Question 2.

Edit the transformation for point P′ such that: $x' = x + 2$ and $y' = y$

a) Describe the location of point P′ in relation to P.
Answer: Point P′ is two units to the right (translation of 2 units in the positive x direction).

b) Determine the equation for the path of point P′.

Answer: $y = (x-2)^2$ or $y' = (x'-2)^2$

### Question 3.

Edit the transformation for point P′ such that: $x' = x$ and $y' = y - 3$

a) Describe the location of point P′ in relation to P.
Answer: Point P′ is three units below point P (translation of 3 units in the negative y direction).

b) Determine the equation for the path of point P′.

Answer: $y' + 3 = (x')^2$ or $y = x^2 - 3$

### Question 4.

Point P is dilated by a factor of 3 away from the $x$ axis, then translated 2 units in the negative $x$ direction. Use your calculator to observe the path of point P′ and determine the equation for $P'(x', y')$ .

Answer: Transformations on $x$ : $x' = 3x - 2$ . Based on the order of operations, the dilation by a factor of 3 will occur first (as per description), followed by the translation of 2 units (in the negative x direction).

Equation: $y' = \frac{(x'+2)^2}{9}$ or $y = \frac{(x+2)^2}{9}$

### Question 5.

Point P is translated by 2 units in the negative $x$ direction, then dilated by a factor of 3 away from the $x$ axis. Use your calculator to observe the path of point P′ and determine the equation for $P'(x', y')$ .

Answer: Transformations on $x$ : $x' = 3(x-2)$ . Parenthesis must be used to order the transformations.

Equation: $y' = \left(\left(\frac{x'}{3}\right) + 2\right)^2$ or $y = \left(\frac{x}{3} + 2\right)^2$ Note that this can also be written as: $y = \frac{1}{9}(x+6)^2$



Author: P. Fox

## Question 6.

Based on your answers to Questions 4 and 5, does the order of transformations matter?

Answer: Yes, the equations are very different. As the point is translated first the dilation 'from' the y axis is accentuated.

## Question 7.

$P(x, y)$ is transformed such that $x' = x$ and $y' = 2y$, use your calculator to observe the path of point P′.

a) Determine the equation for $P'(x', y')$.

Answer: Equation: $\frac{y'}{2} = (x')^2$ or $y = 2x^2$

b) Write an equivalent transformation, based on your equation in part (a).

Answer: The equation shows that this is equivalent to a dilation away from the y axis by a factor of $\frac{1}{\sqrt{2}}$.

## Question 8.

$P(x, y)$ is transformed such that $x' = x - 3$ and $y' = -y$, use your calculator to observe the path of point P′.

a) Determine the equation for $P'(x', y')$.

Answer: Equation: $-y' = (x' + 3)^2$ or $y = -(x+3)^2$

b) State the corresponding transformations.

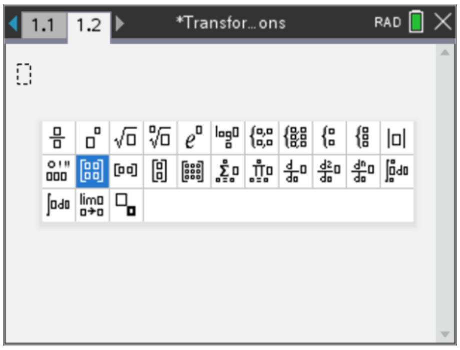
Answer: The graph $y = x^2$ has been reflected in the x axis and translated 3 units in the negative x direction.

## Extension

Transformations can also be described using matrices.

Insert a Calculator Application into your transformation document.

The matrices templated can be found in the maths templates, notice the other matrices templates adjacent to this 2 x 2 templated highlighted opposite.

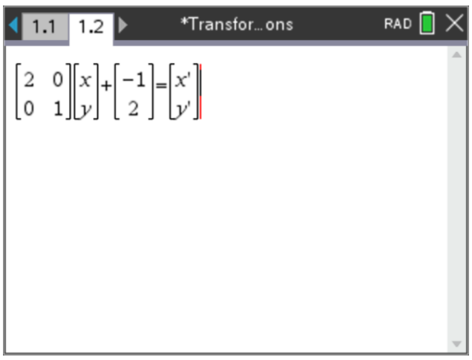



Enter the following equation using the corresponding matrix templates.

Note that the "primed" symbol: ′ can be obtained from the [?!▸] flyout menu.

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Press [enter] to see the calculator result.



**Question 9.**

Use the calculator result to determine the equation for the 'matrix' transformations when applied to point $P(x,y)$ where $y=x^2$.

Answer: From the calculator: $2x-1=x'$ and $y+2=y'$ therefore $y'-2=\left(\frac{x'+1}{2}\right)^2$ which can be written as: $y=\frac{1}{4}(x+1)^2+2$

**Question 10.**

Use the calculator result to determine the equation for the 'matrix' transformation:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x' \\ y' \end{bmatrix}$ when applied to point $P(x,y)$ where $y=x^2$.

Answer: From the calculator: $y=x'$ and $x=y'$ therefore $x'=(y')^2$ which can be written as: $y^2=x$ which can be entered as a relation. [Rotation of 90° or π/2]

**Question 11.**

Use the calculator result to determine the equation for the 'matrix' transformation:

$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x' \\ y' \end{bmatrix}$ when applied to point $P(x,y)$ where $y=x^2$.

Answer: From the calculator: $x+y=x'$ [Eqn1] and $y-x=y'$ [Eqn2]. One option here is to calculate:

Eqn1 – Eqn2: $x'-y'=2x$ therefore: $x=\frac{x'-y'}{2}$

Eqn2 + Eqn1: $x'+y'=2y$ therefore: $y=\frac{x'+y'}{2}$

By substitution into $y=x^2$ results in: $\frac{x'+y'}{2}=\left(\frac{x'-y'}{2}\right)^2$

Expanding and simplifying results in the relationship: $x^2+y^2-2xy-2x-2y=0$

This can be graphed as a relation. While the original parabola has been rotated by 45° or π/4, it has also been dilated.

**Teacher Notes:** The dilation can be demonstrated by using the Analyse > Conics tool in the Graphs menu. Display the focal point for each parabola (the directrix can also be displayed). The simple option is to show that the focal point has moved away from the origin where the turning point on the parabola is located.

The dilation to make question 11 a rotation only was omitted to keep the numbers as simple as possible to encourage students to do some of the algebra by hand.

2D Rotation Matrix: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$



Author: P. Fox