Proportions in Stories

Mathematics Objectives

- Students will construct a data table, analyze the scatter plot of the data, develop an equation to describe the data, and graph the equation to verify that it represents the data.
- Students will compare and contrast proportional and non-proportional linear relationships in real-life stories represented by a table, an equation, and a graph.
- Students will identify characteristics of proportional relationships by analyzing a table, equation, and graph.
- Students will analyze tables, equations, and graphs to determine if two quantities are in a proportional relationship.

Vocabulary

- proportional
- non-proportional
- slope
- *y*-intercept
- constant rate of change
- ratio

About the Lesson

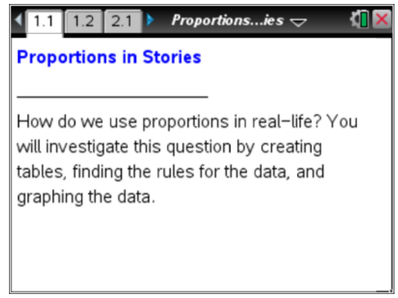
- This lesson involves inputting data into a table, observing the data on coordinate plane, and inputting an equation that corresponds with the table of values.
- As a result, students will:
  - Analyze three real-life situations, two that represent proportional relationships and one that represents a non-proportional linear situation.
  - Determine characteristics of proportional relationships given in multiple representations, such as verbal, numerical, algebraic, and graphical.

TI-Nspire™ Navigator™ System

- Send out the *Proportions_in_Stories.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Proportions_in_Stories_Student.pdf
- Proportions_in_Stories_Student.doc

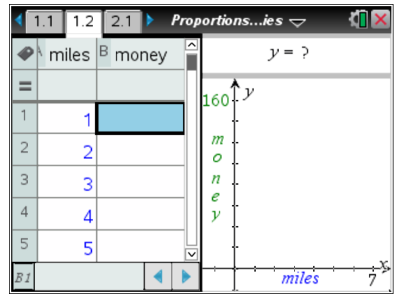
TI-Nspire document

- Proportions_in_Stories.tns

Discussion Points and Possible Answers

Move to page 1.2.

1. You plan to participate in a fund-raising event to collect money for research to fight cancer. Your parents will donate $20 for each mile you walk or run.

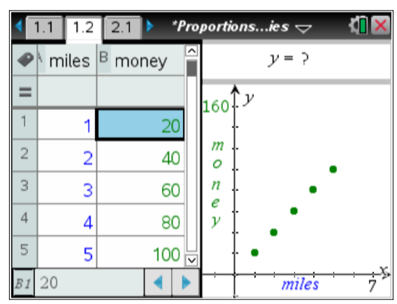


   a. Using the table below, record the total amount of money you can raise depending on how many miles you will walk or run. Show how you determined the amount of the donation by recording the process you used for calculating the amount in the column labeled "Process."

   Answer:

Miles (x)	Process	Money (y)
1	**1 × $20**	**$20**
2	**2 × $20**	**$40**
3	**3 × $20**	**$60**
4	**4 × $20**	**$80**
x	**x × $20**	**20x$**

   b. Input the money in the table on Page 1.2. Describe how the scatter plot is a representation of the table.



   Sample Answer: All of the points are evenly spaced and are located along a positively sloped line. For each increase of a mile in the table and graph, the money increases by $20.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

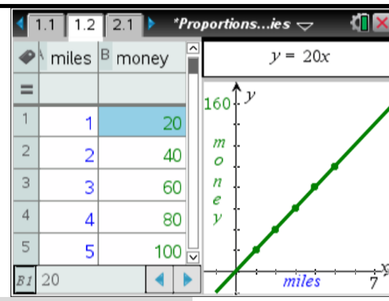
   c. Determine an equation that could be used to calculate the amount of money you can raise if you walk x miles. Make a prediction about the graph of this equation.

   Answer: Let y = the amount of money raised and x = the number of walked miles. Then equation is $y = 20x$. The graph should go through all plotted points.

Teacher Tip: Use patterns in the process column in the table to help students generate the equation. Talk to students about transitions from the specific numbers in the table to a more general equation that can be used for all situations. Model how the slope is related to the rate of change on the table and the graph. Lead students to observe that as they move down from one cell to the next in the table, the numbers increase by 20. As they move from one point to the next point on the graph, they physically go to the right 1 (mile) and up 20 (dollars). (Note: students can also go up 20 (dollars) and then right 1 (mile)). Emphasize that the constant rate of change for this context is $20 per mile. The slope of the graph is 20/1, or 20.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

2. Move the cursor to the y = ? box at the top of Page 1.2 and input the equation.



Tech Tip: As students enter an equation, make sure their pointer is a hand. Students must double click on the ? and then type equation in the text box to replace the ?.

Tech Tip: To enter an equation, students should double tap in the equation bar and then enter the equation using the keyboard that appears.

a. Does your equation represent the data you entered? Explain your reasoning.

<u>**Sample Answer:**</u> The equation represents the data because the line passes through all plotted points.

b. In what way is the $20 per mile donation represented in the data table? How is it represented in the equation? How is it represented in the graph of the equation?

<u>**Answer:**</u> In the data table, $20 is the amount of money added for each mile walked. It is also a ratio of the money raised to the walked miles. (For example, 20:1 = 40:2 = 20.) In the equation, it

is a number by which *x* (the number of miles walked) is multiplied, i.e. the coefficient of *x*. In the graph of the equation, it is the slope of the line (or rate of change: as *x* increases by 1, *y* increases by 20).

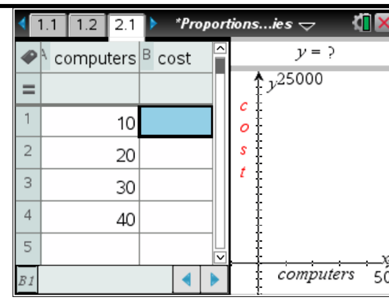
c. Does this situation represent a proportional relationship? Why or why not?

Sample Answers: Yes, it is proportional because the ratio of the money to the miles is the same for all data in the table. In addition, the data in any two rows of the table represent a proportion; for example, 2:5 = 40:100, the points are all on the line that goes through the origin, and the equation has a form $y = mx$.

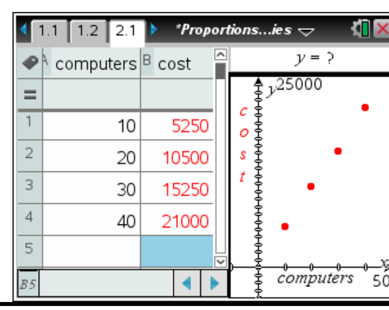
Teacher Tip: Encourage students to provide explanations based on the data table, scatter plot, equation of a line, or the graph of the equation. Have students determine the constant of proportionality.

TI-Nspire Navigator Opportunity: *Quick Poll and Live Presenter*
See Note 2 at the end of this lesson.

Move to page 2.1.



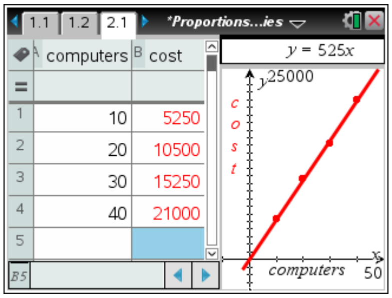
3. Your school plans to buy new computers at $525 per computer. The total number of computers that can be purchased depends on the amount of money in the budget allocated for the computers. Using the table on Page 2.1, record the cost of computers depending on the number of computers.



a. Determine an equation that could be used to budget *x* computers. Input the equation on Page 2.1.

Answer: Let *y* = cost of computers and *x* = number of computers. Then equation is $y = 525x$.

TI-Nspire Navigator Opportunity: Class *Capture*
See Note 1 at the end of this lesson.



Teacher Tip: Help students to make a connection to the previous problem by analyzing patterns in the table and in the points on the graph to generate the equation. Model how rate of change is related to the slope, asking, "If adding 10 computers raises the cost by $5,250, then how much is the cost raised by adding one computer?"

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

b. Does your equation represent the data you entered? Explain your reasoning.

Answer: The equation represents the data because the line passes through all plotted points.

c. In what way is the price of a computer represented in the data table? How is it represented in the equation? How is it represented in the graph of the equation?

Answer: In the data table, 525 is the ratio of the cost of computers to the number of computers. (For example, 5250:10 = 10500:20 = 525.) In the equation, it is the number by which *x* (the number of computers purchased) is multiplied, i.e. coefficient of *x*. In the graph of the equation, it is the slope of the line (or rate of change: as *x* increases by 1, *y* increases by 525).

d. Does this situation represent a proportional relationship? Why or why not?

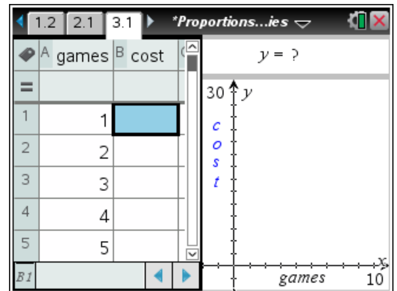
Sample Answers: Yes, because the ratio of the cost to the number of computers is the same for all data in the table; the data are all on the line that goes through the origin; the equation has a form $y = mx$.

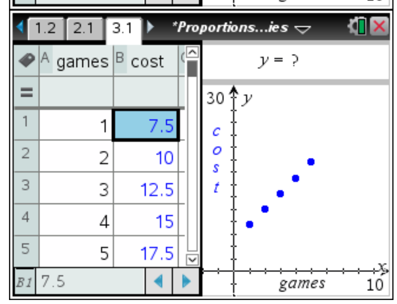
Teacher Tip: Encourage students to provide explanations based on the data table, scatter plot, equation of a line, or the graph of the equation. Have students determine the constant of proportionality.

TI-Nspire Navigator Opportunity: *Quick Poll and Live Presenter*
See Note 2 at the end of this lesson.

Move to page 3.1.

4. You are planning a birthday party at a bowling alley. The shoe rental is \$5 and each game costs \$2.50. Record the cost per person for a different number of games in the table on page 3.1.





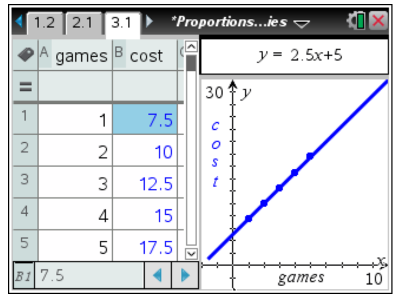
a. Determine an equation that could be used to calculate the cost per person for *x* games. Input the equation on Page 3.1.

<u>**Answer:**</u> Let *y* = cost per person and *x* = number of games. In addition to the games, you have to pay \$5 for the shoe rental. Then equation is $y = 2.50x + 5$.

Teacher Tip: Ask students to compare this situation with the previous two. Make sure they notice that there is a constant cost associated with the shoe rental that does not depend on the number of games you play and that has to be added to the cost of the games in order to find total cost. Discuss this as a one time cost and have students consider how this cost is represented on the table, graph, and equation.

TI-Nspire Navigator Opportunity: *Quick Poll and Live Presenter*
See Note 2 at the end of this lesson.

b. Does your equation represent the data you entered? Explain your reasoning.



Answer: The equation represents the data because the line passes through all plotted points.

c. In what way is the cost per game represented in the data table? How is it represented in the equation? How is it represented in the graph of the equation?

Answer: In the data table, it is amount of money added for each played game. In the equation, it is the number by which *x* (the number of games played) is multiplied, i.e. the coefficient of *x*. In the graph of the equation, it is the slope of the line (or rate of change: as *x* increases by 1, *y* increases by 2.5).

d. In what way is the shoe rental price represented in the data table? How is it represented in the equation? How is it represented in the graph of the equation?

Answer: in the data table, it is a difference between the total cost and how much it will cost to play the game. (For example, for one game it is 7.50 – 2.50 = 5.00.) The cost of the shoe rental is added when one begins to play. In the equation, it is a number that is added to 2.5*x*, and on the graph it is *y*-intercept.

> **Teacher Tip:** The shoe rental fee of $5 is not explicitly shown in the data table. Point student's attention to the fact that the cost of the first game is $7.50, while the cost of each additional game is $2.50. This will help them to identify where the shoe rental price is included in the data table.

e. Does this situation represent a proportional relationship? Why or why not?

Answer: No. Because the ratio of the cost of playing to the number of games is not the same for different numbers of games; or the line does not go through the origin, or the equation is not in the form of $y = mx$.

> **Teacher Tip:** Ask students what makes this relationship non-proportional. Point their attention to the fixed cost of shoe rental that does not depend on the number of games. The fixed cost "shifts" the line up from the origin.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

5. Compare and contrast the data tables, graphs, and equations for the three stories you explored.
   a. What is similar and what is different for the three tables?

   Answer: In all three tables, the values increase by a constant amount; thus, the rate of change is constant for all three situations. In the first two stories, the ratio $\frac{y}{x}$ remains the same for all values. However, in the third story the ratio $\frac{y}{x}$ is different for different values. The ratios of $\frac{y}{x}$ are all the same in proportional relationships, but they are different in non-proportional relationships.

   b. What is similar and what is different for three graphs?

   Answer: All three graphs are linear with a positive slope and a constant rate of change. In the first two stories, the line passes through the origin. In the third problem, the line does not pass through the origin.

   c. What is similar and what is different for three equations?

   Answer: The first two equations are in the form $y = mx$ and represent a proportional relationship. The third equation is in the form of $y = mx + b$ and the relationship is not proportional because after multiplying by a constant there is a constant value that is added. Proportional relationships are multiplicative relationships.

6. Describe the properties of data in a table that represent a proportional relationship.

   Answer: A proportional relationship will ALWAYS reflect a constant ratio of the *x*- and *y*-values.

7. Describe the properties of a graph that represents a proportional relationship.

   Answer: A proportional relationship will ALWAYS be represented by a straight line passing through the origin.

8. Describe the properties of an equation that represents a proportional relationship.

   Answer: A proportional relationship can always be written as an equation in the form of $y = mx$, where m is the constant rate of change. In other words, in a proportional relationship, y will ALWAYS be equal to x being multiplied by the same constant.

Wrap Up

As a result of this lesson, teachers should ensure that students are able to understand:

- Whether a relationship is proportional or non-proportional based on a table, graph, or equation.
- How to approach a situation using a variety of representations and how to describe the relationship between those representations.
- How to identify proportional and non-proportional relationships in real-life situations.

Assessment

An appropriate assessment for this lesson would be for students to create a story representing a proportional situation. Students will create the table, equation, and graph for their situation.

> **Teacher Tip:** Suggest students create their situations in a new TI-Nspire document or add pages to the current document. Then use Live Presenter to let students share their stories and explanations.

TI-Nspire Navigator

Note 1

Name of Feature: Class Capture

Use *Class Capture* to monitor student work when they are completing the table. Have students analyze tables and discuss any differences in their tables. Give students an opportunity to correct their tables after group discussion.

Note 2: Live Presenter

Use *Live Presenter* for question 1.c and have students show and discuss what happens when they move down one cell in the table. They also should be able to show and discuss what happens as they move on the graph. Similarly, for question 3.a students can model the situation using *Live Presenter* and engage the whole class in discussion of how rate of change is related to slope.

Note 3: Quick Poll

Quick Poll: Yes or No – "Is this a proportional relationship?" can be used for questions 2.c, 3.d, and 4.e. Use this as a vehicle for student discussion. Formatively assess the level of student understanding of constant rate of change and how it is related to the table, equation, and graph. Use *Live Presenter* to give control to different students to explain why the relationship is proportional (problems 1 and 2) or non-proportional (problem 3)