Math Objectives

- Students will be able to describe how sampling a population with *N* objects works.
- Students will be able to use the complement of an event to determine probability of an event not occurring.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

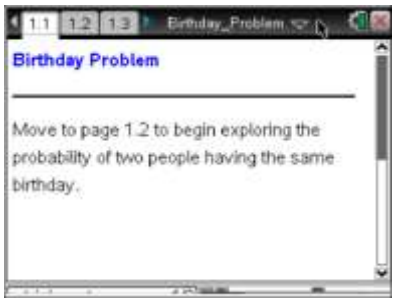
- event
- series
- probability

About the Lesson

- This lesson involves investigating the probability of two people having the same birthday in a crowd of a given size. Topics covered include basic probability theory, sampling distributions and infinite series approximations.
- As a result, students will:
  - Be able to use probability to determine the likelihood of two people having the same birthday in a crowd of a given size.
  - Be able to justify the probability of two people having the same birthday.

TI-Nspire™ Navigator™ System

- Screen Capture and Live Presenter can be used to monitor student progress and allow students to share their answers.
- The student TI-Nspire document file could be sent with TI-Navigator to effectively begin the lesson.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a slider
- Enter data in a spreadsheet

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- On Graphs page, you can retrieve the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Birthday_Problem.pdf

Birthday_Problem.doc

TI-Nspire document

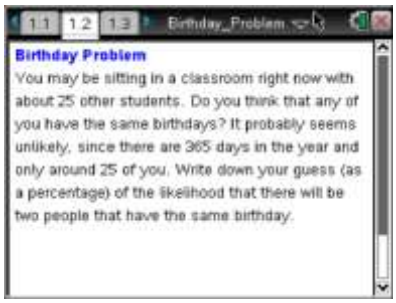
Birthday_Problem.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: *Transfer*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. The Birthday Problem refers to the probability that in a set of randomly chosen people, some pair of them will have the same birthday.

   a. What if someone offered to bet you that any two people in your math class had the same birthday? Would you take the bet?

   Answer: Students should immediately jump to the idea that it depends on how many students are in the class. They should recognize that as the number of students increases, it becomes more likely to have a match.

   b. If there were only one other person in your math class, would you be surprised to find out that they had the same birthday as you? Explain.

   Answer: Students will probably say they would be surprised, as only about 1 in every 365 people will share their birthday.

2. Suppose you are in a classroom of 25 students**.** How likely do you think it is that two of the students in this class have the same birthday? It probably seems unlikely, since there are 365 days in the year and only 25 students. Write down your guess (as a percentage) of the likelihood that there will be two people that have the same birthday.

   Answer: Answers may vary. The discussion within the classroom should focus on how the probability increases as the number of students in the class increases. Many students may immediately understand that the probability is quite high, while others will need to complete this activity to gain a full understanding.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

3. Make a conjecture about the probability of having at least one birthday match in a class of 25 students.

Answer: Answers will vary. The actual probability is greater than .5 and should be somewhat surprising for students. Question 4 further explores this issue.

4. To solve the birthday problem, we need to use one of the basic rules of probability: the sum of the probability that an event *will* happen and the probability that the event *won't* happen is always 1. (In other words, the chance that anything might or might not happen is always 100%.) If we can work out the probability that no two people will have the same birthday, we can use this rule to find the probability that at least two people will share a birthday. Try this process:

P(event happens) + *P*(event doesn't happen) = 1, so,
P(two people share birthday) + *P*(no two people share birthday) = 1, and
P(two people share birthday) = 1 – *P*(no two people share birthday).

a. Assuming 365 days in a year, what is the probability that two people will not share a birthday?

Answer: The answer here should help students develop a better understanding of the problem. Understanding that the first person can have any birthday, the second person's birthday has to be different. Assuming 365 days in a year, all 365 are open for the first person and 364 are open for the second. Divide the "open" days by the total possible days to find the probability of a unique birthday for each student. Then, multiply to find the probability for both:

$$\frac{365}{365}\cdot\frac{364}{365}=1\cdot\frac{364}{365}=\frac{364}{365}$$

The probability of a unique birthday for two students is $\frac{364}{365}$.

b. What is the probability that three or four people will all have different birthdays?

Answer: Building on 4a, there are 363 birthdays out of 365 open for the third person. To find the probability that all three students have unique birthdays, we have to multiply:

$$\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}=1\cdot\frac{132{,}132}{133{,}225}\approx .9918\ .$$

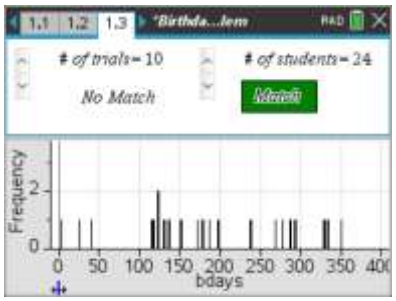
If we want to know the probability that four students will all have unique birthdays, we multiply again: $\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\frac{362}{365}=1\cdot\frac{47{,}831{,}784}{48{,}627{,}125}\approx .9836$.

c. Using this same process, what is the probability of NO birthday matches in a class of 25 students?

Answer: Using the same process from part b, the students should end up with the idea that the formula looks something like $\frac{365!}{(365-n)!\cdot 365^n}$. Naturally, students are not expected to fully develop the formula at this point, but accept any correct discrete variation of the formula that leads to an answer that the probability is close to .43. This should be an opportunity to discuss the complement of the event and discuss that since the probability of no two students having a matching birthday in a class of 25 is .43, then the probability of at least two students sharing a birthday is therefore .57.

TI-Nspire Navigator Opportunity: *Screen Capture or Live Presenter*
See Note 3 at the end of this lesson.

Move to page 1.3.



5. Page 1.3 contains a simulation of number of trials and a frequency distribution of the students that have a matching birthday. Set the number of students at the number of students in your class by pressing the slider up or down. The number of trials represents the number of independent classrooms of that size that you surveyed looking for at least one birthday match. You are not actually asking birthdays, just simulating. For example, if there are 25 students in the class, the number of students should be set at 25. You may conduct as many trials as you like. After pressing the arrow for a simulation, record on paper whether there was a match or no match. Press ctrl . to reset the number of trials.

a. Record the number of trials with a birthday match.

Answer: Answers for each student will vary, but will probably be close to half of the total number of trials.

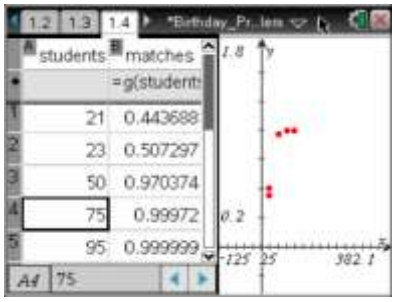
b. Using your results from part a, calculate the probability of two students in your classroom having the same birthday.

Answer: Answers will vary based on the information gathered in part a. The probability of a match can be computed by hand and should mirror that of the formula developed in problem 4. For example, if a student sets the class size at 20, then conducts 20 trials and has 9 matches, the probability (from this simulation) of a match is $\frac{9}{20}$, or .45.

Teacher Tip: Page 1.4 contains a spreadsheet using the formula for calculating the probability of having two birthdays match for a given number of students. Students should experiment with different numbers of students to ascertain the probability of a match.

Move to page 1.4.

6. Page 1.4 uses the formula you found in question 4 to calculate the probability that two students in your class will have the same birthday given the number of students in the class. Type in the number of students to see the probability of two students having a match.



a. How many students need to be in a class to find a probability of a match of more than .50?

Answer: 23 students. Students should experiment with varying numbers of students to determine the answer.

b. How many students need to be in a class to find a probability of a match of more than .99?

Answer: 57 students. Students should experiment with varying numbers of students to determine the answer.

c. Is there anything surprising about the probability of two students having the same birthday from the simulation you conducted?

Answer: Students will probably find it surprising that the number of students needed for a match is so relatively low, just 23 students for a probability > 50% and 57 students for a probability > 99%.

Teacher Tip: As an extension, you could ask the students to discuss the shape of the graph and conjecture to the type of regression model needed to model the data. The growth is logistic.

7. Is it possible to have a probability of a match be 100% or 1? Explain your reasoning.

   Answer: Yes. If there are 366 or more students, at least two must share a birthday because there are only 365 possible birthdays.

Wrap Up

Students should be able to discuss the idea of the probability of having a match of two birthdays for a given number of students. They should also have an understanding of the concept of how a series is needed to calculate the complement of the probability of getting a match.

TI-Nspire Navigator

Note 1

Beginning of lesson, *File Transfer*: If available, you can send the file to students using Navigator. If not you may use Teacher Software to distribute the TI-Nspire document file.

Note 2

Questions 2 and throughout lesson, *Quick Poll (Open Response):* Send a *Quick Poll* to have students answer any of the questions, starting with question 2. As students are answering, discuss how the answers might be different and why.

Note 3

Page 1.3, *Screen Capture* or *Live Presenter*: Starting with page 1.3, use *Screen Capture* or *Live Presenter* to make sure students understand how to use the file. *Screen Captures* of page 1.4 can also help students visually see the possible graph.