Math Objectives

- Students will recognize that the measure of an angle is independent of the distance between any two points on the rays that form the angle and of the length of the representation of the rays.
- Students will relate the measure of an angle to the fraction of a turn in a complete circle.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

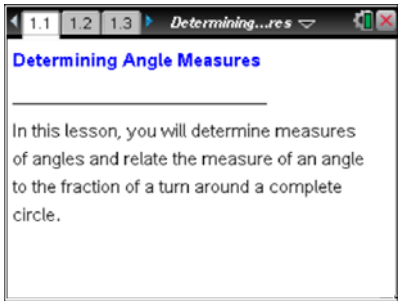
- angle
- ray
- circle
- radius
- degree
- arc

About the Lesson

- This lesson involves angle measures formed by two rays and fractions of a circle.
- As a result, students will:
  - Move points on rays that form an angle and note that the angle measure remains constant as the distance between two points on the rays changes.
  - Extend the representation of the rays so that they appear to be longer and again note that the angle measure is independent of the length of the representation of the sides. The emphasis is on helping students understand that the measure of an angle is determined by the rotation of a side of the angle and not on the linear distance between the two sides of the angle.
  - Move points onto designated spots along one of the angle's rays to create circles with different radii. Students will observe that the arc is the same fraction of a circle no matter how long the radius of the circle is.
  - Describe the measure of the angle in degrees given the fraction of a complete turn.

TI-Nspire™ Navigator™ System

- Screen Capture



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

- Determining_Angle_Measures_Student.pdf
- Determining_Angle_Measures_Student.doc

TI-Nspire document

- Determining_Angle_Measures.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

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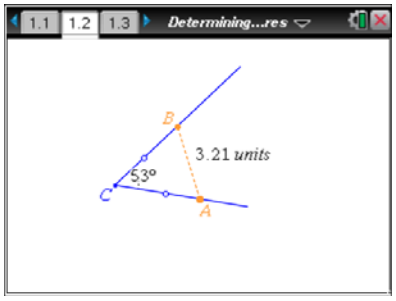
- Quick Poll

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☜) getting ready to grab the point, not a hand pointing at the point (☝). Press ctrl ☜ to grab the point and close the hand (✊).

Move to page 1.2.



1. Move point *A*. Press esc to let go of point *A*. What changes and what stays the same?

   Answer: The angle measure stays the same. The segment gets longer. $\overrightarrow{CB}$ and $\overrightarrow{CA}$ appear to get longer. Students may say that the "sides" of the angle get longer.

   Teacher Tip: Teachers should remind students that a ray has infinite length, even though it looks like it is getting longer. Note that segment $\overrightarrow{AB}$ is not measuring the distance between the 2 lines. This file is set up to measure only angles that measure less than or equal to 180º. You may want to note this for students and discuss the reflex angle.

2. Move the cursor to where $\overrightarrow{CA}$ looks like it ends. Press ctrl ☜ to grab the ray and pull it away from point C. Press esc to let go. What happens to the angle?

   Answer: The angle measure stays the same. The segment stays the same. The ray appears to get longer.

3. Move the arrow to where $\overrightarrow{CB}$ looks like it ends. Grab the ray and pull it away from point *C*. Press esc to let go.
   a. What happens to the angle?

      Answer: The angle measure stays the same. The segment stays the same. The ray appears to get longer.

b. Make a conjecture about what you need to do to change the measure of the angle.

Answer: Answers might include: make the sides farther apart, or one of the angle's rays or sides needs to rotate for the measure of the angle to change.

Teacher Tip: If students suggest making the sides farther apart, ask what they mean by "farther apart." How would you make that happen? If they suggest that the "space" between the sides has to increase, point out that this seemed to happen when they moved the ends of the rays.

You may want to discuss the idea of rotation and point out that you can rotate either side to change the measure of the angle.

4. Grab and move one of the open circles. Press esc to let go of one open circle before grabbing the other open circle.
   a. What is the difference between what happened now and what happened in questions 1 and 2?

Answer: The measure of the angle got larger or smaller. In questions 1 and 2, the angle's measure did not change.

Teacher Tip: Students can move either of the open circles.

   b. What affects the measure of an angle? Explain.

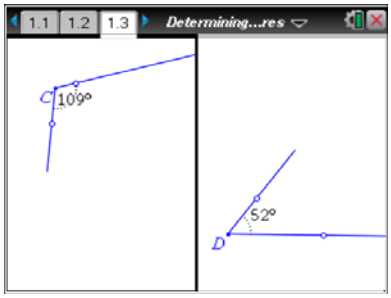
Answer: The measure of an angle is the amount of rotation of one side from the other side of the angle. An explanation might be: If you stand at the vertex looking down one ray, the measure of the angle is the fraction of a complete circle you have to turn to see down the other ray.

   c. What does not affect the measure of an angle? Explain.

Answer: The angle's measure does not depend on the length of $\overline{AB}$ or the length of any segment that connects the two rays. An angle's measure depends on how far one ray or side of the angle is rotated either clockwise or counterclockwise away from the other ray or side.

Teacher Tip: Teachers may need to refer students to question 1 and the .tns file to make the connections between what changes and what does not.

Move to page 1.3.



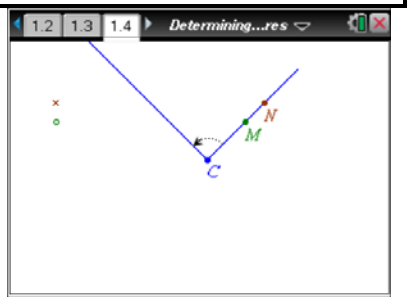
5. Create pairs of angles that are congruent by rotating the rays using the open circles. Describe the conditions you think are necessary to have congruent angles.
   Note: To get from one part of the screen to the other part of the screen, click ctrl tab.

Answer: For angles to be congruent, the two rays must be rotated away from each other by the same amount.

Teacher Tip: Remind students that the orientation does not affect congruence. Any two angles with the same measure will be congruent.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

Move to page 1.4.



An angle is measured in degrees. One degree is $\frac{1}{360}$ of a circle. A complete turn or rotation in a circle measures 360°.

6. Move the open circle to point *M*. The highlighted arc is $\frac{1}{4}$ of a full turn in the circle. How many degrees does this turn represent? How do you know?

Answer: This turn represents 90°. I know because $\frac{1}{4}$ of 360 = 90.

Teacher Tip: You may need to explain the term arc if this is students' first exposure to this term. An arc is an unbroken part of a circle consisting of 2 points on the circle called endpoints and all the points of the circle between them. For these purposes, the arc will be the smaller portion of the circle (the minor arc).

7. Move the *x* to point *N*.
   a. What fraction of a full turn does the highlighted arc represent?

      Answer: It represents $\frac{1}{4}$ of a turn.

   b. How does this compare to your answer in question 6?

      Answer: It is the same as the amount of turn in question 6.

      Teacher Tip: Be sure students recognize that the fraction of the turn and the degree measure are the same in questions 6 and 7. Point out that although the arc length or distance between the radii of the circle (the part that is in bold) is longer for the circle going through point N, the amount of turn to complete a full circle is the same for both circles. You would walk farther but still be only a quarter of the way around the circle.

8. Sammy clicked on a point between point *C* and point *M*. He said his angle represented a smaller angle than when he clicked on point *M*. Do you agree with Sammy? Why or why not?

   Answer: I do not agree with Sammy because the size of the arc does not determine the measure of the angle. The measure of the angle is the amount of turn compared to the full circle. Sammy would have a tiny circle so he would not have to walk very far to be one-fourth of the way around the circle.

   Teacher Tip: If students are struggling, you may have them grab and drag point C to see that the size of the arc does not determine the measure of the angle. To reinforce the fact that the length of the representation for the sides of the angle does not affect the measure of the angle, you might ask students to identify the radii of the different circles and consider whether longer radii create larger angles.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

9. Decide whether each of the statements below is true or false. Explain your reasoning.

	Statements	T or F	Explanation
1.	The measure of an angle depends on how long the sides look.	F	Angle measure is not determined by how long the sides of the angle appear to be.
2.	The measure of an angle depends on the length of a segment connecting the two rays.	F	The angle's measure does not depend on the length of $\overline{AB}$. An angle's measure depends on how far 1 ray or side of the angle is rotated either clockwise or counterclockwise away from the other ray or side.

Teacher Tip: Students may need to look back at question 6 for help.

3.	A half turn around a circle is 180°.	T	180 is $\frac{1}{2}$ of 360.
4.	The measure of an angle depends on the fraction of a complete rotation in any circle centered at the vertex of the angle.	T	Degrees are determined by the fraction of a turn of 360.
5.	An angle whose measure is 60° represents $\frac{1}{3}$ of a complete rotation in a circle.	F	$\frac{1}{3}$ of $360 \neq 60$, it equals 120.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 3 at the end of this lesson.

10. In your own words, explain what determines the measure of an angle.

Sample Answer: An angle's measure is determined by the rotation of one side away from the other, or the measure of an angle is determined by the fraction of the turn in a complete circle.

Wrap Up

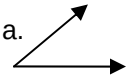
Upon completion of the discussion, the teacher should ensure that students understand:

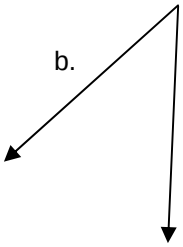
- The measure of an angle does not depend on the distance between two points on the "sides" of the angle nor on the length of the sides.
- The measure of the angle, in degrees, is a fraction of a turn in a complete circle.

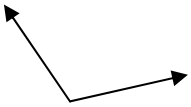
Assessment

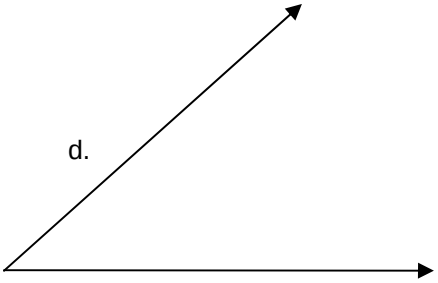
Put the following angles on the board or hand them out on a small sheet of paper. Three of the angles should have the same measure but different orientations.

Order the angles according to their size.

a. 

b. 

c. 

d. 

Answer: a = b = d; c.

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Note 1

Question 5, Screen Capture: Use Screen Capture so that students can see a variety of pairs of congruent angles with different orientations.

Note 2

Question 8, Quick Poll: Send students the following Open Response Quick Poll.

How many degrees does $\frac{1}{6}$ of a full turn in the circle represent?

Answer: 60°

Note 3

Question 9, Quick Poll: Use True/False Quick Polls to collect students' responses to the statements in question 9. Discuss the class results.