Particle Motion 1

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will analyze the relationship between the motion of a particle along a straight line and the graph of the position function that is piecewise linear.
- Students will analyze the relationship between the graph of the position function and the graph of the velocity function.
- Students will analyze the relationship between the graph of the velocity function and the cumulative distance traveled by the particle.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

- position
- cumulative distance
- magnitude
- velocity
- particle motion

About the Lesson

- This lesson involves the motion of a particle along a straight, horizontal line.

As a result, students will:

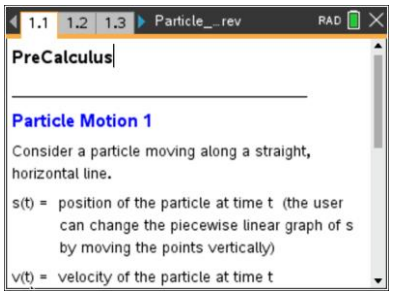
- Analyze the graph of a particle's position function based on numerical information about its position, velocity, and cumulative distance traveled.
- Analyze the motion of the particle based on the graph of its velocity function, and determine the velocity of the particle by finding the slope of the position function.
- Compute the area of rectangles to discover how the graph of the velocity function is related the cumulative distance the particle travels.

TI-Nspire™ Navigator™ System

- Send out the *Particle_Motion_1.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Particle_Motion_1_Student.pdf
- Particle_Motion_1_Student.doc

TI-Nspire document

- Particle_Motion_1.tns.

Discussion Points and Possible Answers

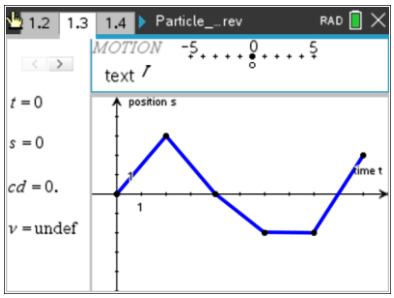
Tech Tip: If students experience difficulty dragging a point, make sure they have not selected more than one point. Press esc to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word point appears. Then select ctrl to grab the point and close the hand (✊). When finished moving the point, select esc to release the point.

Tech Tip: On Page 1.2, the calculator function reset() can be used to reset the time to $t = 0$: type reset() and select enter. This function will also reset the position of the particle to 0 in the top pane. You can also grab and move the open circle on the horizontal axis (calculator pages 1.3 and 1.4) to reset the time to $t = 0$.

- For t measured in seconds, $s(t)$ is the position of the particle at time t, $v(t)$ is the velocity of the particle at time t, and $cd(t)$ is the cumulative distance traveled by the particle from time $t = 0$.
- The values of t, s, cd, and v are given in the left panel on Page 1.3 and Page 1.4.
- The slider arrows are used to change the value of t.
- The graph of s is displayed on Page 1.3, and the graph of v is shown on Page 1.4. The motion of the particle along the horizontal line is modeled in the top panel on both pages.

Note: The points on the graph of s can be grabbed and moved vertically.

Move to page 1.3.



1. Use the slider arrows to change the value of t or grab and move the open circle on the horizontal axis. Observe the relationship among the position of the particle (s), the cumulative distance traveled (cd), and the velocity of the particle (v). Complete the following table.

Answer:

t	0	1	2	3	4	5	6	7	8	9	10
s	0	2	4	2	0	-1	-2	-2	-2	0	2
cd	0	2	4	6	8	9	10	10	10	12	14
v	undef	2	undef	-2	undef	-1	undef	0	undef	2	undef

2. To explore how the motion of the particle along a horizontal line is related to the position graph s, begin by setting $t = 0.2$. As you slowly move the point t to the right, watch how the particle moves along the horizontal line at the top of the page.

   a. On what time intervals is the particle moving to the right? Describe the behavior of the graph of s on these intervals.

   Answer: The particle is moving to the right on the time intervals $(0, 2)$ and $(8, 10)$. The graph of s is increasing on these intervals.

   b. On what time intervals is the particle moving to the left? Describe the behavior of the graph of s on these intervals.

   Answer: The particle is moving to the left on the time interval $(2, 6)$. The graph of s is decreasing on this interval.

   c. Are there any time intervals on which the particle is stationary? If so, describe the behavior of the graph of s on these intervals.

   Answer: The particle remains stationary on the time interval $(6, 8)$. The graph of s is a horizontal line segment, or constant, on this interval.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

3. To explore how the position graph s is related to the velocity graph v, begin by setting $t = 2$ seconds. For each time interval, find the velocity of the particle, the slope of the corresponding line segment of the graph of s, and the direction of the particle motion (left or right).

Answer:

Time Interval	Velocity	Line Segment Slope	Direction
$(0, 2)$	2	2	Right
$(2, 4)$	-2	-2	Left
$(4, 6)$	-1	-1	Left
$(6, 8)$	0	0	Stationary
$(8, 10)$	2	2	Right

a. Make a conjecture about how the velocity of the particle is related to the slope of the position graph.

Answer: The velocity of the particle is equal to the slope of the corresponding line segment of the graph of s.

b. Make a conjecture about the sign (positive or negative) of the velocity and the direction of particle motion (left or right).

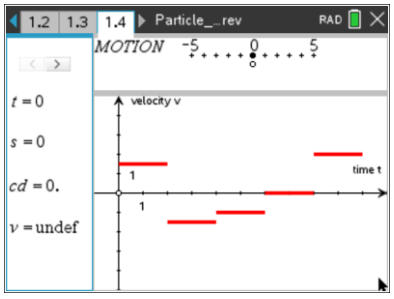
Answer: The particle is moving to the right whenever the velocity is positive and the particle is moving to the left whenever the velocity is negative.

c. How would you change the graph of s in order for the magnitude of the velocity to increase? Decrease? Grab and move some of the points on the graph of s to check this conjecture.

Answer: If the graph of s is steeper, then the magnitude of the velocity will be greater. If the graph of s is flatter, then the magnitude of the velocity will be smaller. Students might connect this to the absolute value of the slope of the line segment. The greater the absolute value of the slope of the line segment, the greater the velocity of the particle. The smaller the absolute value of the slope of the line segment, the smaller the velocity of the particle.

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 2 at the end of this lesson.

Move to page 1.4.



4. Explore the relationship between the graph of the velocity, the cumulative distance, and the position of the particle.
   a. What is the cumulative distance at time $t = 2$ seconds? Find the area of the rectangle bounded by the graph of v and the horizontal axis over the interval $(0, 2)$. How do these two values compare?

Answer: The cumulative distance is 4. The area of the rectangle is $A = (2)(2) = 4$. These two values are the same.

b. Find the area of the rectangle bounded by the graph of v and the horizontal axis over the interval $(2,4)$.

- Add this area to the area found in part (a). How does this sum compare to the cumulative distance at time $t = 4$?

Answer: Area of the rectangle is $A=(2)(2)=4$. This area is added to the area found in part (a): $4+4=8$. This is also the value of the cumulative distance at time $t = 4$.

- Subtract this area from the area found in part (a). How does this difference compare to the position of the particle at time $t = 4$?

Answer: This area is subtracted from the area found in part (a): $4-4=0$. This is also the position of the particle at time $t = 4$.

c. Find the area of the rectangle bounded by the graph of v and the horizontal axis over the interval $(4,6)$.

- Add this area to the cumulative distance at time $t = 4$. How does this sum compare to the cumulative distance at time $t = 6$?

Answer: Area of the rectangle is $A=(2)(1)=2$. This area is added to the cumulative distance at time $t=4$: $8+2=10$. This is the cumulative distance at time $t = 6$.

- Subtract this area from the position of the particle at time $t = 4$. How does this difference compare to the position of the particle at time $t = 6$?

Answer: Subtracted from the position of the particle at time $t=4$: $0-2=-2$. This is the position of the particle at time $t = 6$.

d. Explain how to use the graph of v to find the cumulative distance at times $t = 8$ and $t = 10$ and find these values.

Answer: Find the area of each rectangle formed by the graph of the velocity function and the horizontal axis. Add these areas to find the cumulative distance.
Cumulative distance at time $t=8$: $(2)(2)+(2)(2)+(2)(1)+(2)(0)=10$
Cumulative distance at time $t=10$: $(2)(2)+(2)(2)+(2)(1)+(2)(0)+(2)(2)=14$

e. Explain how to use the graph of v to find the position of the particle at times $t = 8$ and $t = 10$ and find these values.

Answer: Find the area of each rectangle formed by the graph of the velocity function and the horizontal axis. Find the position by adding (to 0, the initial position) areas associated with positive velocities and subtracting areas associated with negative velocities.
Position at time $t = 8$: $(2)(2) - (2)(2) - (2)(1) + (2)(0) = -2$
Position at time $t = 10$: $(2)(2) - (2)(2) - (2)(1) + (2)(0) + (2)(2) = 2$

Teacher Tip: The particle is stationary, or does not change position, whenever the velocity is 0. The area of the associated rectangle is 0.

f. Explain why the cumulative distance is the sum of areas (associated with the graph of the velocity function) but the position of the particle is dependent upon the sign of the velocity (or signed area).

Answer: The cumulative distance is the total distance traveled by the particle, to the left and to the right. It seems reasonable to add, or total, all of the area associated with the graph of the velocity function. The position of the particle is the difference between distance traveled to the right and distance traveled to the left. Therefore, the position is dependent upon the sign of the various areas.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The relationship between the motion of a particle along a straight line and the graph of the position function.
- The relationship between the graph of a position function (consisting of straight line segments) and the velocity function.
- How to compute the cumulative distance and position of the particle using the velocity function.

Extension

- Consider asking students about these relationships if the graph of s is a smooth curve.
- Consider the velocity of the particle at time 2, 4, 6, and 8. Ask students to explain why the velocity is undefined at these values of t.

Assessment

- Give students the graph of a position function made up of line segments. Ask them to construct a graph of the velocity function. Use the velocity function to find the cumulative distance traveled at certain times.
- Give students an initial position for a particle and the graph of its velocity function. Ask them to draw the position function.
- Ask students to create a velocity function for a particle which moves left, then right, and then left again back to its original position.
- Use Class Capture to discuss student responses.

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Note 1

Question 2, Name of Feature: Class Capture

Ask students to create a position graph so that a particle moves left for 4 seconds, then moves right for 4 seconds, and stays stationary for 2 seconds. Use Class Capture to compare and contrast student responses.

Note 2

Question 3, Name of Feature: Class Capture and Live Presenter

Use Class Capture and Live Presenter with your class to verify this answer. Challenge students to create a graph of s with specific velocities.