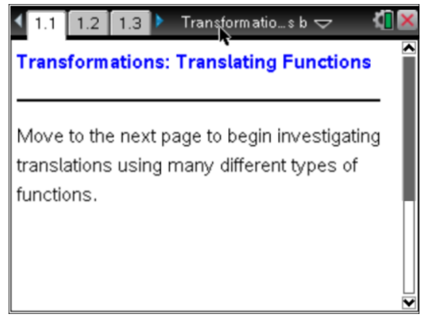
Transformations: Translating Functions



Math Objectives

- Students will vertically translate a function by adding a constant and write the appropriate symbolic representation for the translated function.
- Students will horizontally translate a function by adding a constant and write the appropriate symbolic representation for the translated function.
- Students will identify the effect of a and b in $y = f(x - a) + b$ on the graph of a general function $y = f(x)$.
- Students will look for and make use of structure (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- transformations
- horizontal/vertical translations

About the Lesson

- This lesson involves translating different types of function graphs using sliders.
- As a result, students will:
  - Understand how adding a constant outside the function moves the graph of a function up and down.
  - Discover how adding a constant inside the function moves the graph of a function left and right.

TI-Nspire™ Navigator™ System

- Use File Transfer to share the TI-Nspire document file.
- Use Screen Capture or Live Presenter to monitor student progress or have them share their investigations.
- Use Quick Poll to assess student understanding.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

- Transformations_Translating_Functions_Student.pdf
- Transformations_Translating_Functions_Student.doc

TI-Nspire document

- Transformations_Translating_Functions.tns

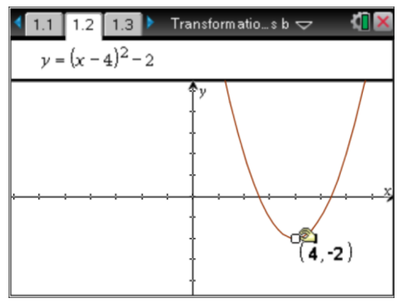
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: ***File Transfer***
See Note 1 at the end of this lesson.

Tech Tip: Page 1.2 is designed to easily allow students to translate the function graph. Instruct the students to move the cursor to the vertex until they get the open hand (✋) and click the touchpad or press [enter]. The point should then slowly blink. Then the point can move by pressing the directional arrows of the touchpad.

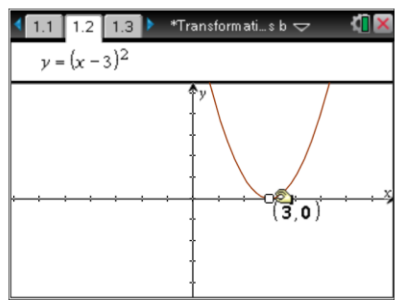
Move to page 1.2.



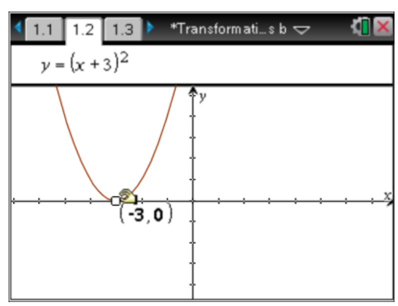
1. Grab and drag the open point identified by the coordinates. Recall that the vertex form of a parabola is $f(x) = (x - h)^2 + k$ and that the vertex of a parabola is (h, k).

   a. What changes in the function as you move the graph along the *x*-axis? What changes as you move it along the *y*-axis?

   Answer: When the graph is moved horizontally, the constant inside the parentheses changes. When the graph is moved vertically, the constant outside the parentheses changes. When the function is shifted vertically, the *x*-coordinate of the vertex is added outside the parentheses. When the function is shifted horizontally, the *y*-coordinate is subtracted inside the parentheses.



   b. Move the vertex to (3, 0). What is the equation of the graph? Then move the vertex to (–3, 0). What is the equation of the graph now? What is the relationship between the horizontally shifted graphs and the vertex?



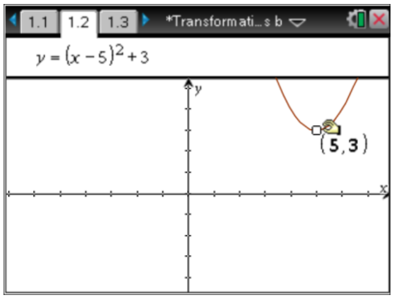
   Answer: When the point is at (3, 0), the equation is $y = (x - 3)^2$. When the point is at (–3, 0), the equation is $y = (x + 3)^2$. The *x*-coordinate is subtracted inside the parentheses of the equation (Note: Adding 3 is equivalent to subtracting –3 in this case).

Teaching Tip: The coefficient *a* was intentionally left out of the vertex form in question 1 because this activity only deals with translations. The activity Transformations: Dilating Functions deals with that concept. If students ask about it, feel free to discuss it, but be aware that it is not a part of this activity.

Teaching Tip: Students often struggle to understand why the *x*-coordinate of the vertex is subtracted inside the parentheses when the graph is shifted horizontally. The common version of the vertex form of a quadratic equation is $y = (x - h)^2 + k$. However, rewriting it in the equivalent form $y - k = (x - h)^2$ can help show that both of the constants work as subtractions when applied to the related variable.

TI-Nspire Navigator Opportunity: ***Screen Capture or Live Presenter***
See Note 2 at the end of this lesson.

2. Observe the function as you move the vertex point around in the first quadrant. Notice that the values of *h* and *k* are both positive in the first quadrant.



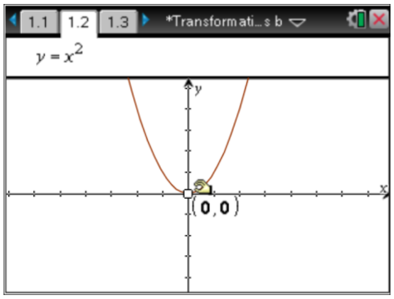
   a. What is the relationship between the vertex *y*-coordinate *k* and the value that is being added (outside the parentheses) in the function?

   <u>Answer:</u> The *y*-coordinate is added outside the parentheses.

   b. What is the relationship between the vertex *x*-coordinate *h* and the value that is being added (inside the parentheses) in the function?

   <u>Answer:</u> The *x*-coordinate is subtracted inside the parentheses.

3. Move the vertex of the parabola back to the origin. What is the equation on the screen, and why does it appear that way?



   <u>Answer:</u> They returned to zero and the equation is now $y = x^2$. This is the simplified form of $y = (x - 0)^2 + 0$.

Tech Tip: All of the following pages are designed to easily allow students to translate the function graph. Instruct the students to move the cursor to the *h* or *k* until they get the open hand (☝) and click the touchpad or press [enter]. The point should then slowly blink. Then the point can move by pressing the directional arrows of the touchpad.

Move to page 1.3.

4. Make observations about the coordinate points as the value of *k* changes.
   a. Each function has one point identified by an ordered pair. Record the coordinates of this point for different values of *k* for each function. Click ▲ or ▼ on the left side of the screen to see the next function.

   Answers:

k	***a*(*x*)**	***b*(*x*)**	***c*(*x*)**	***d*(*x*)**
3	(0, 3)	(–2, 0)	(0, 4)	(0, 4)
0	(0, 0)	(–2, –3)	(0, 1)	(0, 1)
–3	(0, 3)	(–2, –6)	(0, –2)	(0, –2)

   b. Looking at the data that you recorded in the table, what effect does changing the value of *k* have on the function graph? Does it work the same way for all of the functions?

   Answer: It changes the *y*-coordinate by the same value added to the function.

Move to page 1.4.

5. Make observations about the identified coordinate points as the value of *h* changes.
   a. Each function has one point identified by an ordered pair. Record the coordinates of this point for different values of *h* for each function. Click ▲ or ▼ in the upper left corner of the screen to see the next function.

   Answers:

h	***t*(*x*)**	***u*(*x*)**	***v*(*x*)**	***w*(*x*)**
3	(4, 1)	(3, –1)	(4, 0)	(2, –1)
0	(1, 1)	(0, –1)	(1, 0)	(–1, –1)
–3	(–2, 1)	(–3, –1)	(–2, 0)	(-4, –1)

b. Looking at the data that you recorded in the table, what effect does changing the value of *h* have on the function graph? Does it work the same way for all of the functions?

Answer: It changes the *x*-coordinate by the opposite of the value inside the function. In other words, the value is the same as the value subtracted inside the parentheses.

Teaching Tip: The graph of *m*(*x*) is a vertically reflected parabola. Students may not recognize this. Even though this concept will be addressed in more detail later, you might consider asking students about what caused the parabola to open downward if time allows.

6. Suppose a function *g*(*x*) has the point (3, 4) on its graph. What translations are needed to move the point to (–2, 1)? What is the equation for this new function in terms of *g*(*x*)?

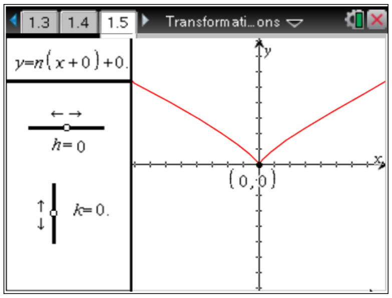
Answer: The graph must be translated 5 units to the left and 3 units down. The resulting equation would be $y = g(x + 5) - 3$.

Teaching Tip: Question 6 can be a challenge for some students. You may want to encourage students to plot the points to help them visualize the transformations.

Move to page 1.5.

7. Make observations as you change the value of *h* and *k*. Write the general form of the *f* function in terms of *h* and *k*.

Answer: The function $y = f(x)$ is translated right and left as *h* is changed and translated up or down a *k* is changed in the equation $y = f(x - h) + k$.



TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Teaching Tip: You may want to challenge students and give them different functions to explore using page 1.5.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to shift the graph of a function vertically using appropriate notation.
- How to shift the graph of a function horizontally using appropriate notation.
- How to combine transformations of a function and use appropriate notation.

TI-Nspire Navigator

Note 1

Before the Activity, *File Transfer*: Use the file transfer to efficiently send the TI-Nspire document to the students. Using TI-Navigator will allow students to receive the file without having to leave their seats or use extra cables.

Note 2

Entire Document, *Screen Capture* or *Live Presenter*: If students experience difficulty with the operation of a file or a question, use the computer software or Live Presenter with TI-Navigator. You can also use this as a way to facilitate student discussion.

Note 3

Question 7, *Quick Poll*: A Quick Poll can be given at the conclusion of the lesson. You can save the results and show the results at the start of the next class to discuss possible misunderstandings students may have.

The following are some sample questions you can use:

1. Which transformation has the graph of $y = f(x - 3)$ undergone from the graph of $y = f(x)$?
   a. Moved up 3
   b. Moved down 3
   c. Moved left 3
   d. Moved right 3

2. If (2, 3) is on the graph of $y = f(x)$, which of the following is on the graph of $y = f(x) + 4$?
   a. (6, 3)
   b. (–2, 3)
   c. (2, 7)
   d. (2, –1)

3. True/False: The domain of a function is left unchanged by a vertical shift. Answer: True

