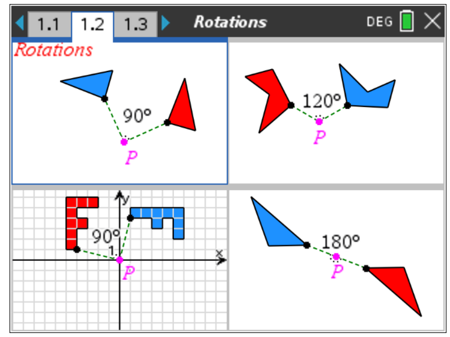
Transformational Geometry Rotations

TEACHER NOTES AND SOLUTIONS

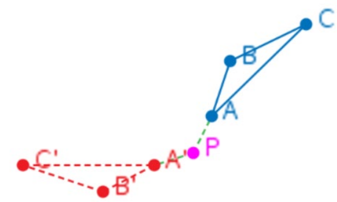


Transformational Geometry is a way to study geometry by focusing on geometric “movements” or “transformations” and observing/studying properties about these figures.

There are four geometric transformations:

< Reflections   < Translations   **< Rotations**   < Dilations

Play - Investigate - Explore - Discover
P I E D



In the figure to the right, ΔABC is rotated about point P.

ΔABC is called the pre-image while $\Delta A'B'C'$ is called the image (of rotation).

$\Delta A'B'C'$ is read “triangle A prime, B prime, C prime.”

Point P is called the point of rotation.

Download and install the red TI-Nspire student software and the Rotations TNS file from the website where you obtained this document.

Then you can interact with these figures, too. If you decide not to download the software, or if you cannot, you can still do this activity along with the videos.

A **conjecture** is an opinion or conclusion based on what is observed.

1. What conjecture(s) can you make based upon what you observed about a triangle and its image after being rotated?

When a triangle is rotated about a point so many degrees, it appears that the pre-image and image triangles are congruent; that each pair of corresponding angles is congruent and each pair of corresponding angles is congruent. The triangles appear to be the same shape and same size.

2 a. When a figure is rotated about a point through an angle of positive measure, the figure rotates in a

counter-clockwise direction.

b. When a figure is rotated about a point through an angle of negative measure, the figure rotates in a

clockwise direction.

3. a) After observing the angle measures being shown, what conjecture can you make?

Corresponding angles have the same measure (are congruent).

b) After observing the side measures being shown, what conjecture can you make?

Corresponding sides have the same length (are congruent).

c) Note: do not say all the angles are equal, or do not say all the sides are equal. They aren't.
The sides and angles that correspond to one another have equal lengths and measures, respectively.

d) What is true about the triangles? State that using symbols.

$\Delta ABC \cong \Delta A'B'C'$

4. a) After observing the perimeters being shown, what conjecture can you make?
Why should this be true?

If a triangle is rotated about a point, then the pre-image triangle and its image triangle have equal perimeters.
This should be true because the triangles are congruent, so corresponding sides have the same length. If you add up the lengths of the 3 sides of each triangle, the sums will be the same number because you are adding the same three numbers.

b) After observing the areas being shown, what conjecture can you make?
Why should this be true?

If a triangle is rotated about a point, then the pre-image triangle and its image triangle have equal areas.
This should be true because the triangles are congruent, so corresponding sides have the same length. The corresponding altitudes (heights) of the triangles will have the same length. So when you multiply one half times the base times the height on each triangle, you are multiplying the same numbers in each case. (provided you are multiplying bases and heights that correspond)

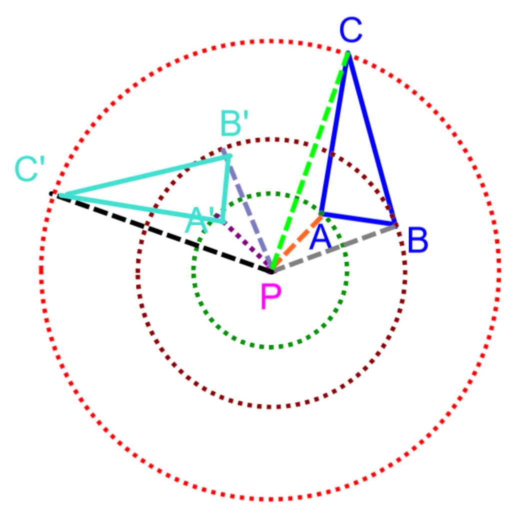
5. After observing how triangles have been rotated about a point, you should be able to rotate a triangle about a point through an angle of 90^o, or a multiple of 90^o. Ideally you will need a compass and straightedge.

a. Rotate Δ ABC about point P 90°.
   Label this image Δ A'B'C'.

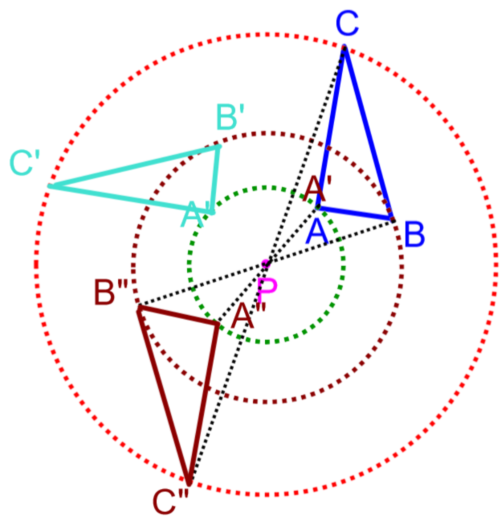
b. Rotate Δ ABC about point P 180°.
   Label this image Δ A"B"C".

c. Rotate Δ ABC about point P 270°.
   Label this image Δ A'"B'"C'".

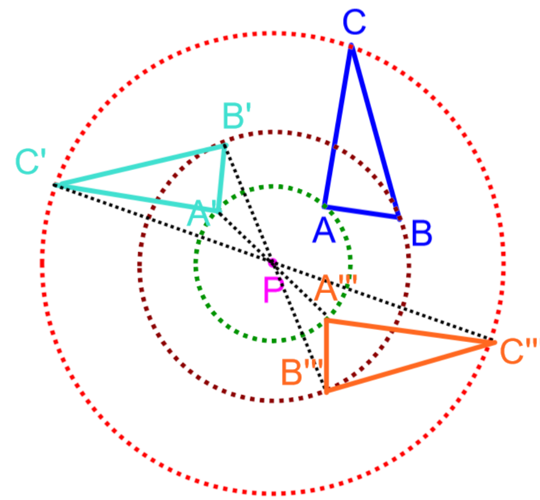
a.



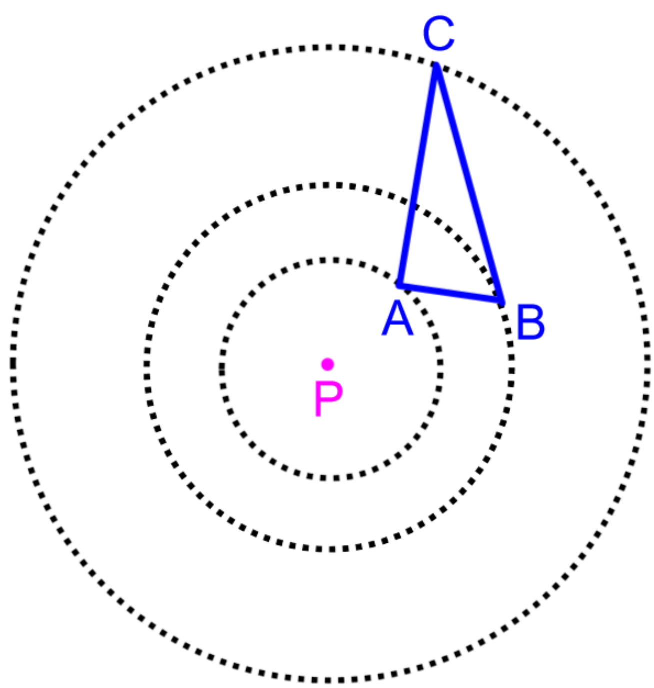
b.



c.



6. If you do not have access to a compass, do problem 5 above using the figure below.
You will need a straightedge (or ruler).
Concentric circles are circles in the same plane, with the same center, but different length radii.



a. Rotate Δ ABC about point P 90°.
Label this image Δ A'B'C'.

b. Rotate Δ ABC about point P 180°.
Label this image Δ A"B"C".

c. Rotate Δ ABC about point P 270°.
Label this image Δ A'"B'"C'".

See solutions for number 5 above.

7. Based on your explorations and observations:
a. What appears to be true about segments

PA and PA'? **equal** PB and PB'? **equal** PC and PC' **equal**

Why should these 3 statements be true?
(hint: look at the results to either #5 or #6 above)
PA and PA' are radii of the same circle, PB and PB' are radii of the same circle, PC and PC' are radii of the same circle. Radii of the same circle are congruent (have the same length) by the definition of a circle.

8. Grids and Coordinates Rotate 90^o Complete the following.

a) If a triangle is rotated 90^o about the origin, the x-coordinate of the image is the

opposite of the y-coordinate of the pre-image triangle.

b) If a triangle is rotated 90^o about the origin, the y-coordinate of the image is the

x-coordinate of the pre-image triangle.

c) Or in symbols, If (x, y) is a point on the pre-image, then $\underline{\underline{(-y, x)}}$ is a point on the image triangle.

$(x, y) \rightarrow \underline{\underline{(-y, x)}}$ where '$\rightarrow$' means 'maps to'

9. Grids and Coordinates Rotate 180^o Complete the following.

a) If a triangle is rotated 180^o about the origin, the x-coordinate of the image is the

opposite of the x-coordinate of the pre-image triangle.

b) If a triangle is rotated 180^o about the origin, the y-coordinate of the image is the

opposite of the y-coordinate of the pre-image triangle.

c) Or in symbols, If (x, y) is a point on the pre-image, then $\underline{\underline{(-x, -y)}}$ is a point on the image triangle.

$(x, y) \rightarrow \underline{\underline{(-x, -y)}}$ where '$\rightarrow$' means 'maps to'

10. Grids and Coordinates Rotate 270^o Complete the following.

a) If a triangle is rotated 270^o about the origin, the x-coordinate of the image is the

y-coordinate of the pre-image triangle.

b) If a triangle is rotated 270^o about the origin, the y-coordinate of the image is the

opposite of the x-coordinate of the pre-image triangle.

c) Or in symbols, If (x, y) is a point on the pre-image, then $\underline{\underline{(y, -x)}}$ is a point on the image triangle.

$(x, y) \rightarrow \underline{\underline{(y, -x)}}$ where '$\rightarrow$' means 'maps to'

11. Grids and Coordinates Rotate 360^o Complete the following.

a) If a triangle is rotated 360^o about the origin, the x-coordinate of the image is the

x-coordinate of the pre-image triangle.

b) If a triangle is rotated 360^o about the origin, the y-coordinate of the image is the

y-coordinate of the pre-image triangle.

c) Or in symbols, If (x, y) is a point on the pre-image, then $\underline{\underline{(x, y)}}$ is a point on the image triangle.

$(x, y) \rightarrow \underline{\underline{(x, y)}}$ where $'\rightarrow'$ means 'maps to'

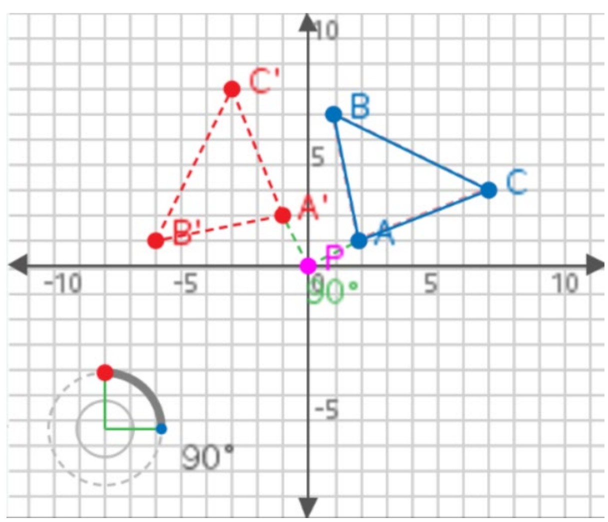
12. A rotation of -90^o is equivalent to a rotation of what positive angle measure? $\underline{\underline{270^o}}$

13. A rotation of -180^o is equivalent to a rotation of what positive angle measure? $\underline{\underline{180^o}}$

14. A rotation of -270^o is equivalent to a rotation of what positive angle measure? $\underline{\underline{90^o}}$

Corresponding Sides

We have already shown that corresponding sides of triangle rotated about the origin are equal in length. What else seems to be true about pairs of corresponding sides? Let's explore.



15. Calculate the slopes of corresponding sides either graphically or by slope formula.

Show your work in the space provided below.

Write all answers as fractions in simplest form.

Record your answers in the #15 row of the table on the bottom of this page.
Look for patterns!

ΔABC is rotated 90^o about the origin.

15. Find the slopes as fractions of:

$a.\ \overline{AB}\ and\ \overline{A'B'}$ $b.\ \overline{BC}\ and\ \overline{B'C'}$ $c.\ \overline{AC}\ and\ \overline{A'C'}$

m(AB) $= -\frac{5}{1} = -5$

m(A'B') $= \frac{1}{5}$

m(BC) $= -\frac{3}{6} = -\frac{1}{2}$

m(B'C') $= \frac{6}{3} = \frac{2}{1} = 2$

m(AC) $= \frac{2}{5}$

m(A'C') $= -\frac{5}{2}$

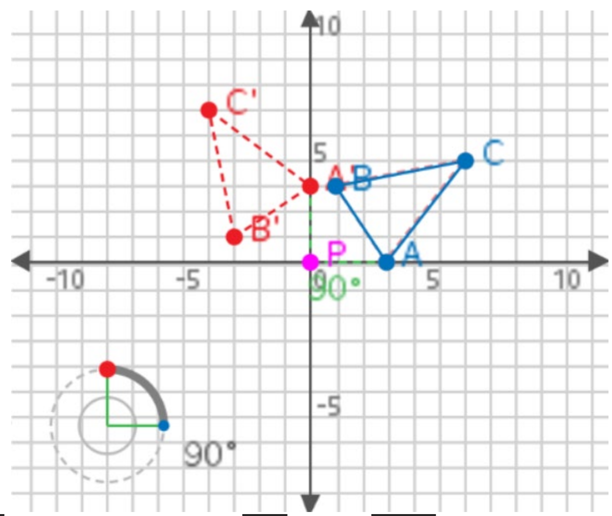
Rotate 90^o	$m(\overline{AB})$	$m(\overline{A'B'})$	$m(\overline{BC})$	$m(\overline{B'C'})$	$m(\overline{AC})$	$m(\overline{A'C'})$
# 15	$-\frac{5}{1}$	$\frac{1}{5}$	$-\frac{1}{2}$	$\frac{2}{1}$	$\frac{2}{5}$	$-\frac{5}{2}$
# 16	$-\frac{3}{2}$	$\frac{2}{3}$	$\frac{1}{5}$	$-\frac{5}{1}$	$\frac{4}{3}$	$-\frac{3}{4}$
# 17 video	$-\frac{2}{1}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{2}{1}$	$\frac{1}{4}$	$-\frac{4}{1}$

16. Calculate the slopes of corresponding sides either graphically or by slope formula.

Show your work in the space provided below.

Write all answers as fractions in simplest form.

Record your answers in the #16 row of the table on the bottom of the previous page.
Look for patterns!



ΔABC is rotated 90^o about the origin.
16. Find the slopes as fractions of:

$a.\ \overline{AB}\ and\ \overline{A'B'}$ $b.\ \overline{BC}\ and\ \overline{B'C'}$ $c.\ \overline{AC}\ and\ \overline{A'C'}$

See the table above.

17. There is an example on the video that we want you to record the slopes of sides into the table on the bottom of the previous page. Pause the video and record the slopes as fractions in simplest form.

See the table above.

18. After completing exercises 15 – 17 and recording the slopes in the table, then do this exercise.
a. Look at the slopes of corresponding sides in the table on the previous page.
What pattern do you notice?

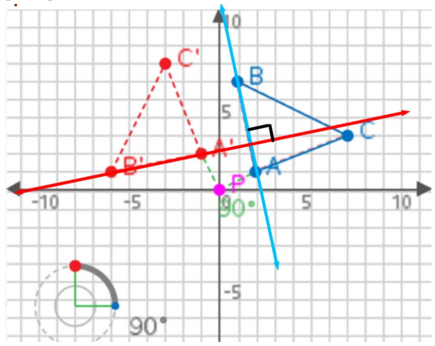
The slopes of corresponding sides are opposites and reciprocals of each other.
Or the product of their slopes is negative one, – 1.

b. What does that mean is true about lines: $\overleftrightarrow{AB}\ and\ \overleftrightarrow{A'B'}$? $\overleftrightarrow{BC}\ and\ \overleftrightarrow{B'C'}$? $\overleftrightarrow{AC}\ and\ \overleftrightarrow{A'C'}$?

Each pair of these lines are perpendicular to each other.

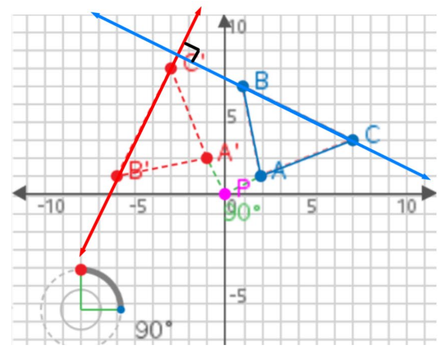
$\overleftrightarrow{AB} \perp \overleftrightarrow{A'B'}$

slope $-\frac{5}{1}$ $\quad \frac{1}{5}$



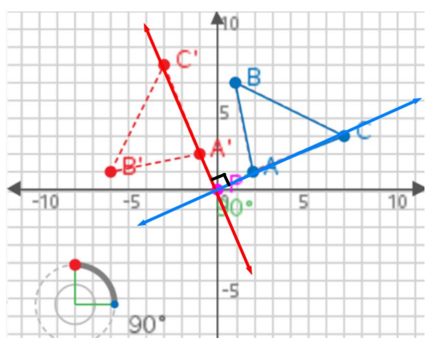
$\overleftrightarrow{BC} \perp \overleftrightarrow{B'C'}$

slope $-\frac{1}{2}$ $\quad \frac{2}{1}$



$\overleftrightarrow{AC} \perp \overleftrightarrow{A'C'}$

slope $\frac{1}{2}$ $\quad -\frac{2}{1}$



The exercise to the right was not included in the student sheets. However, we recommend that you ask your students to establish why this is true. The proof is shown here. Even if only a few understand it.

How could we prove that lines through corresponding side are always perpendicular?

Suppose that the coordinates of two vertices on the pre-image are

(a,b) and (c,d)

Then the slope of the line through the pre-image is:

$$m = \frac{d-b}{c-a}$$

Since the triangles are rotated 90 degrees,

(a,b) ⟶ (-b, a) (c, d) ⟶ (-d, c)

The slope of the line through the image vertices is:

$$m = \frac{c-a}{-d-(-b)}$$

$$m = \frac{c-a}{-(d-b)}$$

$$m = -\frac{c-a}{d-b}$$

reciprocals of each other!

and opposites of each other

Which makes the lines perpendicular to each other!

Enjoy…