

Volume of a Watermelon

Student Activity

7 8 9 10 11 12



TI-Nspire™



Activity



Student

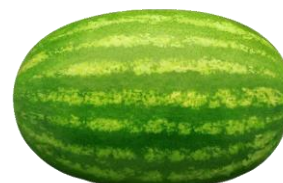


50 min

Introduction

This activity looks at three different ways to measure the volume of a watermelon.

- Archimedes' principle (measurement by displacement of water)
- Approximation – Cutting the watermelon into slices that can be approximated by cylinders
- Calculus – Determining an equation to model the shape of the watermelon and rotating it around an axis.



Part 1 - Archimedes

For this measurement you will need:

- Watermelon
- Bucket – big enough to submerge your watermelon*.
- Measuring jug
- Water

*If you don't have a bucket big enough, try one that will hold half your watermelon.

Archimedes' principle states:

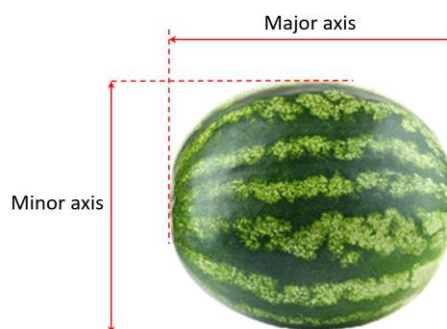
A body at rest in a fluid is acted upon by a force pushing upward called the buoyant force, which is equal to the weight of the fluid that the body displaces. **If the body is completely submerged, the volume of fluid displaced is equal to the volume of the body.**

Completely fill the bucket with water. Place the watermelon in the bucket, you will most likely need to nudge the watermelon to ensure it is completely submerged, avoid displacing too much water with your hand. Once the water has settled, remove the watermelon from the bucket, again, try and avoid displacing any additional water. Once the watermelon is removed from the bucket, use your measuring jug to refill the bucket and therefore determine the volume of water that was displaced by the watermelon.

Question: 1.

According to your measurements and Archimedes' principle, what is the volume of your watermelon.

Note: 1 litre = 1000cm³



Part 2 – Preparing your watermelon

For this section you will need:

- Watermelon (same)
- Camera / phone
- Computer + TI-Nspire software + USB cable
- Ruler / Tape measure

It is important that the image be an accurate scaled version of the actual watermelon. Measure and record the major and minor axis lengths carefully.

Step 1:

Photograph your watermelon. Make sure the image is 'square' to the melon. Try and keep the background as plain as possible to make it easier to see the outline once the image has been sent to your calculator.

Step 2:

The calculator screen has a 4:3 aspect ratio, the background image however is mapped to 320:217. To ensure the watermelon doesn't get distorted, use imaging software to crop your watermelon image to the 320:217 ratio. Try and crop as close to the edge (major axis) of your watermelon image. The actual watermelon's dimensions are unlikely to be in the same ratio as your calculator screen, this will most likely leave space at the top and bottom of the image. Don't worry too much about the resolution of your image, the TI software will reduce the image resolution to suit the calculator.

Step 3:

Launch the TI-Nspire software and connect your calculator to the computer. Start a new document, insert a Graphs application and insert your image. You may wish to set the scale and axis location before saving the file to your handheld.



Step 4:

Measure and record the length (major axis) and height (minor axis) of your watermelon.

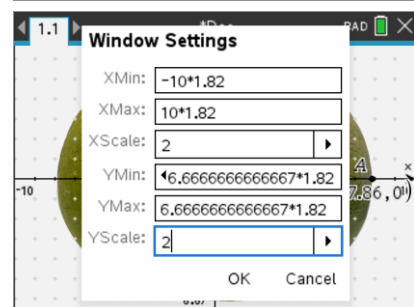
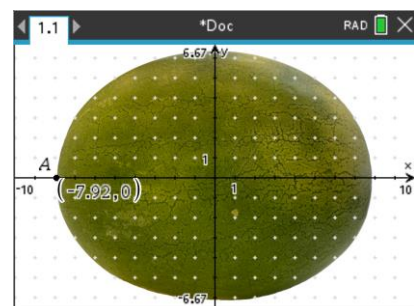
On the calculator, select zoom standard from the window/zoom menu, this will ensure the cartesian plane has the same scale in the x and y direction and that the origin is in the centre of the screen.

Press P and select point by coordinates, enter coordinates that will be close to the left most point of the watermelon, the point can also be adjusted by dragging. Record the coordinates of the point, then move the point to the right most point of the watermelon, once again record the coordinates.

Use the two extreme points and your measurements for the major axis to calculate the zoom factor for your window settings.

The window settings can be changed directly by multiplying each minimum and maximum value by the corresponding scale factor. Similarly, the location of the origin can be changed by dragging the screen or adding or subtracting the corresponding translation to each minimum and maximum value.

To confirm that the scale has been set correctly, move the point again to locate the x coordinates of the extremities of the watermelon. The difference between these extreme values should be equal to the length of the major axis. Do the same for the extremities of the y coordinates for the watermelon, the difference between these values should be equal to the measurement for the minor axis of the watermelon.



Question: 2.

The general equation for an ellipse is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Determine an equation to model the edge of the watermelon storing the corresponding values in 'a' and 'b'.

Note: As the equation is a relation, use Y_1 to model the top of the watermelon and $Y_2 = -Y_1$ to model the bottom.

You may also need to translate your function, alternatively, move the origin to the 'centre' of the watermelon by adding/subtracting values to the Xmin, Xmax, Ymin, Ymax values.

Question: 3.

Your teacher will cut the watermelon into slices such that each slice resembles 'short' cylinder, a cross-section of the watermelon. Use a ruler to measure the dimensions of your cylinder (watermelon slice) and determine the corresponding volume.

**Question: 4.**

Collect the volume of watermelon each slice from each of your classmates and hence determine an approximate volume for the original watermelon.

Question: 5.

Determine the location of **your** slice of watermelon on the graph. Explain how the radius of your slice of watermelon relates to the function.

Question: 6.

Transpose your equation to make y the subject. Check your answer by graphing the equation as $f_1(x)$.

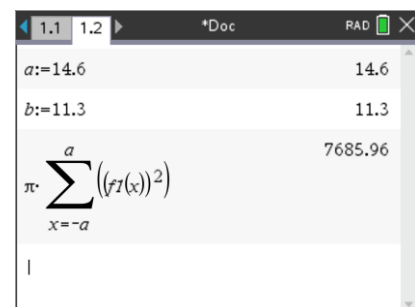
Question: 7.

Explain what this expression is doing:

$$\pi \sum_{-a}^a f_1(x)^2,$$

where 'a' represents the nearest integer value for the x-axis intercepts of the function modelling the watermelon.

Try this on your calculator (see opposite)

**Question: 8.**

Explain how this expression (below) relates to the expression in the previous question.

$$\pi \int_{-a}^a y^2 dx$$

Application

A measuring jug (shown) contains measurements in 100ml increments, up to 500ml.

- Insert a copy of the image into your calculator.
- Scale your calculator window settings accordingly.
- Model the side(s) of the measuring jug using a linear function.
- Rotate the function around the y – axis to form a solid of revolution, and check the approximated volume for the 500ml marking.
- Determine the distance between the 100ml – 200ml – 300ml – 400ml markings.

