

# A Random Piece of Pi

## Student Activity

7 8 9 10 11 12



## Introduction

In this activity you will explore a pattern related to the occurrence of common factors between two randomly selected numbers. Specifically, the investigation looks at the probability that the two numbers will be co-prime.

---

**Co-Prime:** Two numbers are said to be Co-Prime if their highest common factor is 1.

---

### Example:

The numbers 27 and 35 are composite (not prime), however they are co-prime. The highest common factor of 27 and 35 is 1, so they are co-prime.

### Question: 1.

Identify which of the following pairs of numbers are co-prime:

a) (25, 60)

b) (39, 100)

c) (45, 64)

d) (98, 56)

e) (23, 115)

f) (18, 29)

## Useful Commands

### GCD:

GCD = Greatest Common Divisor, also known as the Highest Common Factor. This command can be used to identify the highest common factor of any two numbers.


 > **Number** > **Highest Common Factor**

**Note:** The screen shown has the language set to English (UK). If your calculator is set to English (US) Greatest Common Divisor will be displayed.


Use the GCD command to check your answers to Question 1.

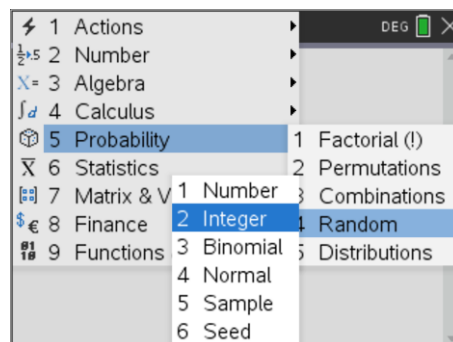
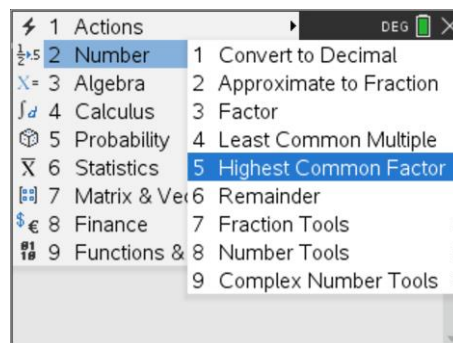
### RandInt:

The random Integer command can generate random whole numbers between two specified values.

 > **Probability** > **Random** > **Integer**

Try: `randint(1,100)`

Press  several times to see what sort of numbers are produced.



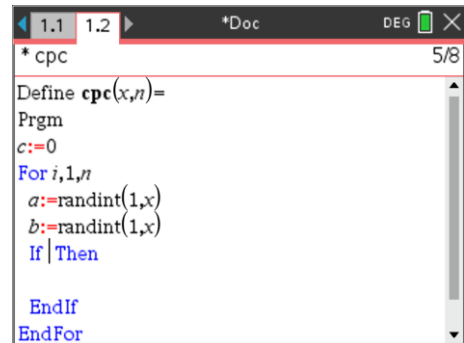
## Writing the Program

The start of a co-prime counting program (cpc) is shown opposite.

In the program x represents the size of the random numbers.

Theoretically this should be infinite, however very larger numbers will slow program execution. Consider the size of x in relation to the number of samples, 'n'.

For example sampling 1000 pairs of random numbers to check if they are co-prime, x should be more than 100 as this limit would provide for only 9900 possible pairs of random numbers.



```

1.1 1.2 *Doc DEG 5/8
*cpc
Define cpc(x,n)=
Prgm
c:=0
For i,1,n
a:=randint(1,x)
b:=randint(1,x)
If | Then
EndIf
EndFor
  
```

Finish writing the co-prime counting program so that you can explore the proportion of randomly selected pairs of numbers are co-prime. Your program needs to:

- Test and record when two randomly selected numbers are co-prime.
- Calculate the proportion of the numbers tested that are co-prime.
- Provide the opportunity to vary the number of tests conducted (n) and the magnitude of the numbers being generated (x)

### Question: 2.

Testing your program is an important step in any coding exercise. Explain how you tested your program to make sure it was working as expected.

### Question: 3.

Run your program several times using `cpc(100,100)`, `cpc(100,1000)` and `cpc(100,10000)`. Record the proportion of co-primes in each sample.

### Question: 4.

Run your program several times using `cpc(1000,100)`, `cpc(1000,1000)` and `cpc(1000,10000)`. Record the proportion of co-primes in each sample.

**Note:** `cpc(1000,10000)` will take a while to run!

### Question: 5.

For each of the proportions computed in the previous question, calculate:

$$\sqrt{\frac{6}{p}} \quad \text{where } p \text{ represents the proportion.}$$

### Question: 6.

Add the calculation from Question 5 to your program and display the result. Run your program several times using the following: `cpc(1000,1000)`. Calculate the average of the results and discuss.