Matrices – Act 2

Answers

7 8 9 10 **11** 12

TI-Nspire Investigation Student 30 min

Multiplication

Start a new document and insert a calculator application.

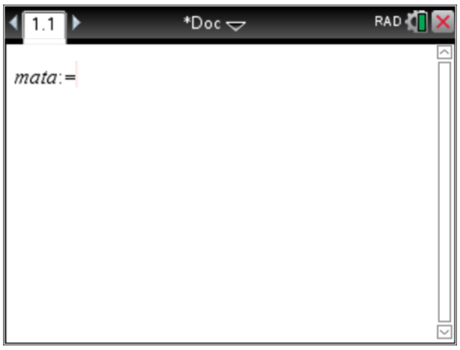
The first matrix will be defined as: *mata*

It is not necessary to have 'mat' at the start of the variable name, however it will help immediately identify which of your defined variables is a matrix.

In this document a matrix is denoted as: [A].

One way to define a variable is to use ":="

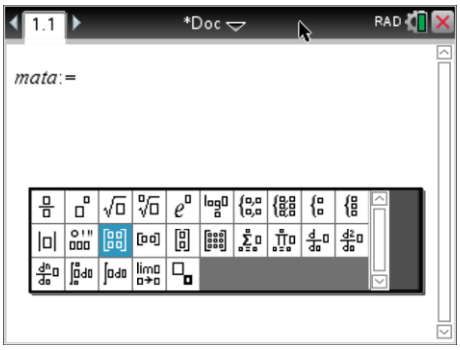
Type: mata:=



Use the maths template and select the 2 x 2 matrix template (shown opposite)

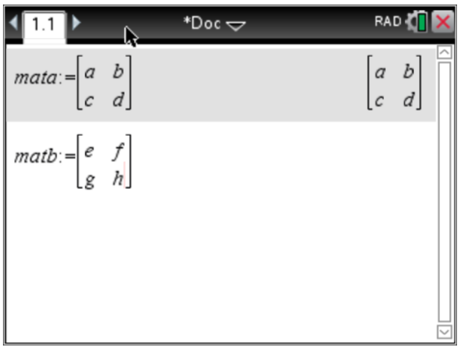
Define matrix A as: mata:= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Use the [**TAB**] key to navigate around the matrix.



Repeat the above process to create a second matrix called Matrix B

matb := $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

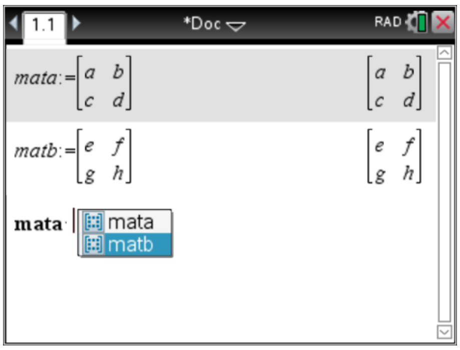


Multiply the two matrices together:

mata × matb

Note:

To avoid typing the variable name again, press the variable button and select the variable name from the list.



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Questions

1. Write down the rule for multiplying two: 2 x 2 matrices and include a diagram showing how each component is determined.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Note: A teacher file (power point slide show) is available from the activity download section of the website. This slide show progressively steps through, step by step, multiplication involving a 2 x 2 pair of matrices. Each step is clearly tracked using colour, animation and where appropriate, text.

2. Use your rule to answer the following: (ie: Do these questions by hand)

a) $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 9 & 17 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 21 & 7 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 6 & 8 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 7 & 10 \end{bmatrix}$

d) $\begin{bmatrix} 6 & 8 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 14 \\ 3 & 3 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

f) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

g) $\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

h) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$

3. Check your answers to the above questions using the CAS calculator.

(Answers above) – Note that some students may answer (a) and (b) the same if they are doing them by hand under the assumption that the commutative law does apply… of course it does not.

Commutative Law:

$$a \times b = b \times a$$

4. Use the algebraic representation of matrix multiplication from Question1 combined with selected answers from Question 2 to determine if the commutative law (above) applies to the multiplication of matrices.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} \text{ vs } \begin{bmatrix} e & f \\ g & h \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{bmatrix}$$

The algebraic expressions are very different so the commutative law does not apply to matrices. Students should however identify that for (e) and (f) in question 3 the order of the matrices does not matter in this special case.

5. Multiply matrix A by itself; mata × mata, check this answer against mata^2.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

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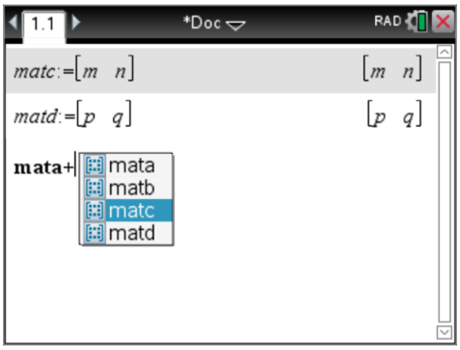
Dimensions

Two new matrices need to be defined: matc and matd

matc : = $\begin{bmatrix} m & n \end{bmatrix}$

matd := $\begin{bmatrix} p \\ q \end{bmatrix}$



6. Explore the multiplication of matrices with different dimensions. Record the results for each of the following:

a) [A] × [D] = $\begin{bmatrix} ap+bq \\ cp+dq \end{bmatrix}$

[2 x 2] x [2 x 1] produces [2 x 1]

b) [C] × [A] = $\begin{bmatrix} am+cn & bm+dn \end{bmatrix}$

[1 x 2] x [2 x 2] produces [1 x 2]

c) [A] × [C] = Dimension error

Can't be multiplied

d) [D] × [A] = Dimension error

Can't be multiplied

e) [C] × [D] = $\begin{bmatrix} mp+nq \end{bmatrix}$

[1 x 2] x [2 x 1] produces [1 x 1]

f) [D] × [C] = = $\begin{bmatrix} mp & np \\ mq & nq \end{bmatrix}$

[2 x 1] x [1 x 2] produces [2 x 2]

g) Comment on your findings with regards to the multiplication of matrices with different dimensions. **Hint**: Write down the dimensions of each matrix "Rows x Columns" next to each multiplication problem above noting which multiplication problems produce a result and the corresponding dimensions of the result.

Answers to Q6 suggest, when the dimensions are written as shown, when the number of columns in the first matrix matches the number of rows in the second, the matrices can be multiplied. Students may also note that the dimensions of the result (answer) are given by the number of rows for the first matrix by the number of columns of the second.

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