## Science Objectives

- Students will measure or calculate the net force on an object.
- Students will describe the relationship between mass, weight, and gravitational acceleration.
- Students will measure or calculate the normal force due to gravity on an object.

## About the Lesson

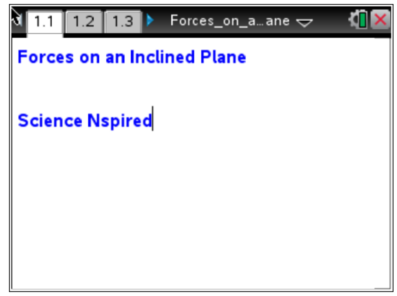
- In this activity, students will collect data on the magnitude of the force acting on an object resting on an inclined plane.
- As a result, students will:
  - Use their data to identify the mathematical relationship between the angle of the plane and the component of gravitational force that is parallel to the plane.
  - Explore the relationship between the angle of the plane and the component of gravitational force that is perpendicular to the plane.

## TI-Nspire™ Navigator™

- Send out the *Forces_on_an_Inclined_Plane.tns* file.
- Monitor student progress using Screen Capture.
- Use Live Presenter to spotlight student answers.

## Activity Materials

- TI-Nspire™ Technology
- mass with string attachment point
- Vernier Dual-Range Force Sensor and EasyLink™ or Go!™Link interface
- piece of wood and several books for making inclined plane
- copy of student worksheet
- string
- pen or pencil
- meter stick or tape measure




**TI-Nspire™ Technology Skills:**

- Download a TI-Nspire document
- Open a document
- Move between pages
- Collect data

**Tech Tip:**

Access free tutorials at [http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials](http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials)

**Lesson Files:**

*Student Activity*

- Forces_on_an_Inclined_Plane.doc
- Forces_on_an_Inclined_Plane.pdf

*TI-Nspire document*

- Forces_on_an_Inclined_Plane.tns

## Discussion Points and Possible Answers

**Teacher Preparation**

Students should be familiar with trigonometric functions and vectors before carrying out this experiment. You may also wish to review with them the relationship among mass, gravitational force, and weight.

A small rolling cart with additional mass in it is ideal as the mass in this activity. If a cart is not available, the mass used should be able to slide down the inclined plane even at low angles. This will increase the accuracy of the collected force data.

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**Classroom Management**

This activity is designed to be student-centered, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students should record their answers to the questions on blank paper.

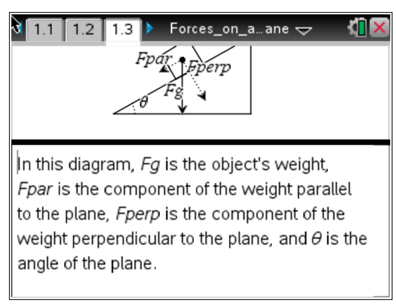
The ideas contained in the following pages are intended to provide a framework for how the activity will progress. Suggestions are also provided to help ensure that the objectives of the activity are met.

In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.

The following questions will guide student exploration during this activity:

- How is the angle of an inclined plane related to the magnitude of the force acting on an object along the inclined plane?
- What mathematical function describes this relationship?

**Part 1: Collecting Force Data**




Students should open the file ***Forces_on_an_Inclined_Plane.tns*** and read the first three pages. and then answer Questions 1 and 2.

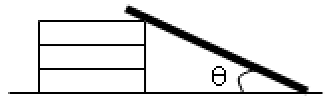
Q1. What value of $\theta$ will produce the largest value of ***Fpar***? What value of $\theta$ will produce the smallest value of ***Fpar***? Explain your answer.

**Answer:** Students' answers will vary.

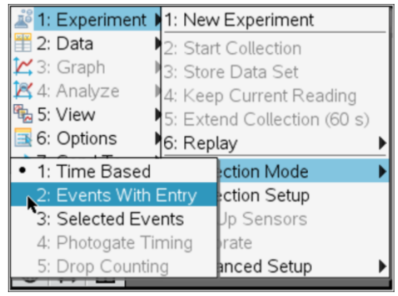
Q2. What mathematical relationships do you think there are between $\theta$, ***Fpar***, ***Fperp***, and ***Fg***? Explain your answer.

**Answer:** Students' answers will vary.

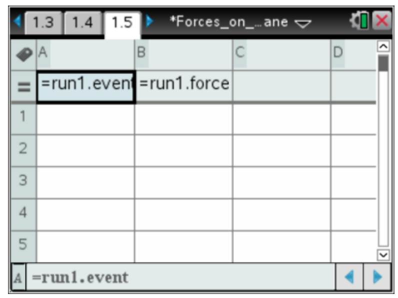
1. First, students should set up the inclined plane and measure its angle of inclination ($\theta$ in the diagram to the right) at several points. Students should either measure the angle of the plane using a protractor or calculate the angle from the length and height of the plane.




2. Next, students should move to page 1.4, which contains an empty *Vernier DataQuest* application. They should connect the force sensor to the EasyLink or Go!Link interface and then connect the interface to their handheld or computer. They should then set up the sensor to Events with Entry mode (**Menu > Experiment > Collection Mode > Events with Entry**).

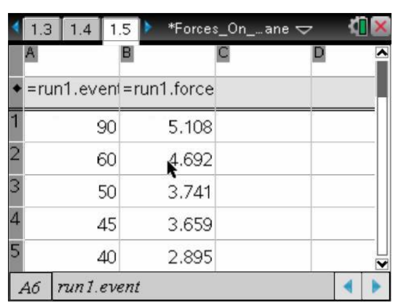



3. Next, students should move to page 1.5, which contains an empty *Lists & Spreadsheet* application. They should insert the variable run1.event in column A and run1.force in column B. To assign data to a column, students should type the name of the variable in the title bar of the column.




4. Next, students should zero the force sensor (**Menu > Experiment > Set Up Sensors > Zero**), return to page 1.4 and begin the experiment.
5. Next, students should use the string to connect the mass to the hook on the bottom of the force sensor. (Make sure the switch on the force sensor is set to the correct range. The range students use will depend on the mass they use.) They should let the mass hang from the force sensor without touching anything. Once the weight reading on the force sensor has stabilized, students should record a data point, using 90 as the event value (because the angle is 90° from the horizontal).
6. Next, students should place the mass on the inclined plane at its highest angle, with the force sensor holding the mass on the plane, as shown to the right. They should record another data point, using the angle of the plane as the event value.

7. Students repeat Step 5, reducing the angle to each of the values they calculated or measured in Step 1. Once data collection is complete, students can stop the data collection, close the data collection box, and disconnect the force sensor. (Note: The data shown here are simulated; students' data will vary.) When they have collected their data, they should return to page 1.5.



Q3. To which of the three forces (***Fpar***, ***Fperp***, or ***Fg***) do the data in column B correspond? Explain your answer.

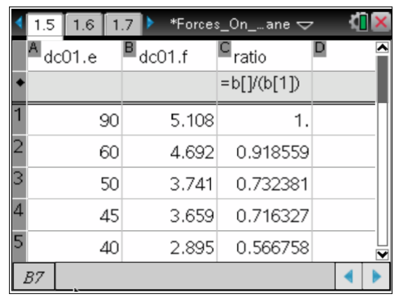
**Answer:** The data in column B correspond to ***Fpar***, the component of the object's weight that acts parallel to the surface of the plane.

Q4. Do your data support your predictions from Question 1? If not, identify the errors in your reasoning that led you to make your predictions.

**Answer:** Students' answers will vary. Encourage metacognitive thinking to help students identify their errors in logic.

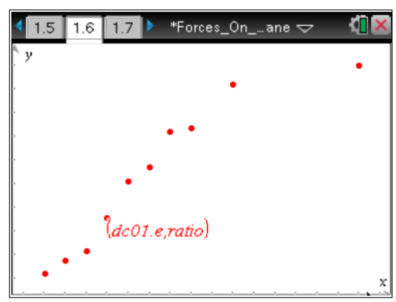
**Part 2: Fitting a Curve to the Data**

1. Next, students calculate the ratio of the force on the mass at each angle to the total weight of the mass. They should use the force value they measured in Step 4 of part 1 as the weight of the mass. They store the ratio in the variable ratio.



| | A dc01.e | B dc01.f | C ratio | D |
|---|---|---|---|---|
| ◆ | | | =b[]/(b[1]) | |
| 1 | 90 | 5.108 | 1. | |
| 2 | 60 | 4.692 | 0.918559 | |
| 3 | 50 | 3.741 | 0.732381 | |
| 4 | 45 | 3.659 | 0.716327 | |
| 5 | 40 | 2.895 | 0.566758 | |

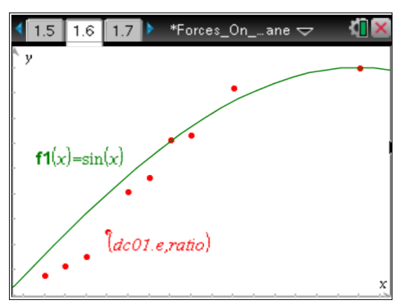
2. Next, students move to page 1.6, which contains an empty *Graphs* page. Students change the graph to a scatter plot and plot ratio vs. angle.




Q5. What functional form appears to fit your data?

**Answer:** Students' answers will vary.

3. Next, students change the graph to a function graph and plot functions that they think will best fit their data. Encourage class discussion of the data and the most appropriate functional forms.

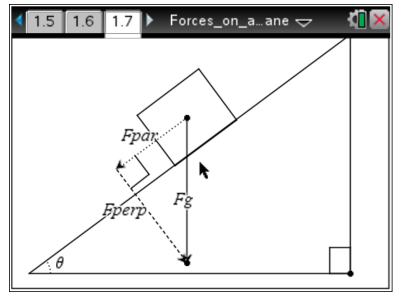



Q6. What function produced the best fit to your data?

**Answer:** Students should make sure the angle setting is set to Degrees (**Menu > File > Document Settings**) before plotting their data. Students' answers will vary, but a sine function should be the best fit (i.e., ratio should be the sine of the angle of the inclined plane). Once students have reached this conclusion, draw a larger version of the diagram on page 1.3 on the board. Work with students to help them see why the ratio is equal to the sine of the inclined plane.

**Part 3: Calculating the Normal Force**

1. Next, students will determine the magnitude of the component of gravitational force that is perpendicular to the plane (***Fperp***). They first examine a larger diagram showing the forces and angles on the object (this diagram is similar to the one on page 1.3).

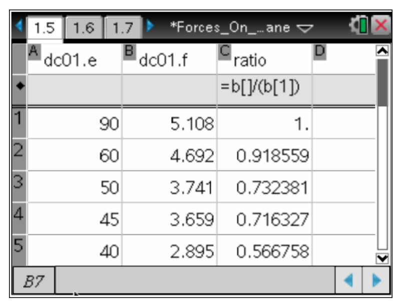



Q7. Write an equation for ***Fperp*** in terms of ***Fpar*** and ***Fg***.

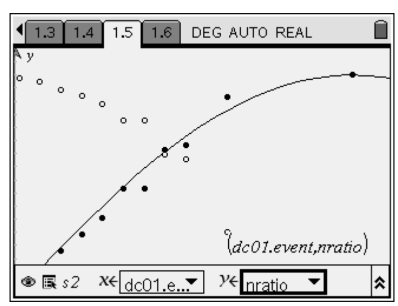
**<u>Answer</u>:** ***Fperp*** and ***Fpar*** are the legs of a right triangle. The hypotenuse of the triangle is ***Fg***. Therefore, the Pythagorean theorem can be used to describe the relationship between ***Fpar***, ***Fperp***, and ***Fg***, as shown below:

$$(F_{par})^2 + (F_{perp})^2 = (F_g)^2$$
$$(F_{perp})^2 = (F_g)^2 - (F_{par})^2$$
$$F_{perp} = \sqrt{(F_g)^2 - (F_{par})^2}$$

2. Next, students move back to the *Lists & Spreadsheet* application on page 1.5 and define the variable ***fperp*** using the equation they wrote in Question 7. They then calculate the ratio of ***fperp*** to the weight of the object. They store this ratio in column E and give column E the variable name ***nratio***.



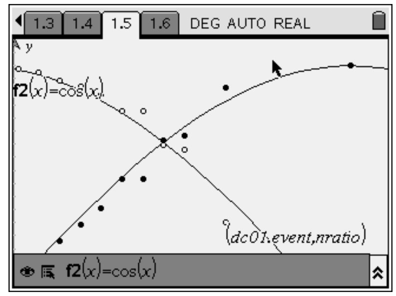

3. Next, students plot ***nratio*** vs. ***angle*** on page 1.6.




Q8. What functional form appears to fit your data?

**<u>Answer</u>:** Students' answers will vary. Encourage students to discuss their answers.

4. Next, students change the graph to a function graph and plot functions that they think will best fit their data. Encourage class discussion of the data and the most appropriate functional forms. Then, students should answer Questions 9–13. If time allows, you may have students repeat the activity using a different mass.




Q9. What function produced the best fit to your data?

**Answer:** Students' answers will vary, but a cosine function should be the best fit (i.e., ***nratio*** should be the cosine of the angle of the inclined plane). Once students have reached this conclusion, draw a larger version of the diagram on page 1.3 on the board. Work with students to help them see why the ratio is equal to the cosine of the inclined plane.

Q10. Were the predictions you made in Question 2 about the relationships between ***Fg***, ***Fpar,*** and ***Fperp*** correct? If not, identify any errors in the reasoning that led you to make your predictions.

**Answer:** Students' answers will vary. Encourage metacognitive thinking to help students identify their errors in reasoning.

Q11. Suppose you calculated the ratio of ***Fpar*** to ***Fperp*** and plotted this ratio versus the angle of the inclined plane. What would the plot look like? Explain your answer.

**Answer:** The graph should resemble a tangent graph. ***Fpar*** and ***Fperp*** are the legs of the right triangle. If time allows, you can have students insert another *Graphs & Geometry* page, calculate the ratio of the two forces, and plot the ratio vs. the angle of the plane to confirm their answers.

Q12. A box with a weight of 8.34 N is resting on an inclined plane. The plane is inclined 13.6° to the horizontal. What is the normal force on the box? What is the force acting parallel to the inclined plane?

**Answer:** The normal force is equal to the product of the weight and the cosine of the angle of the plane, as shown below:

$$F_{perp} = F_g \cos\Theta$$
$$F_{perp} = (8.34\ \text{N})\cos(13.6°)$$
$$F_{perp} = (8.34\ \text{N})(0.9720)$$
$$F_{perp} = 8.11\ \text{N}$$

The force parallel to the plane is equal to the product of the weight and the sine of the angle, as shown below:

$F_{par} = F_g \sin\Theta$

$F_{par} = (8.34\text{ N})\sin(13.6°)$

$F_{par} = (8.34\text{ N})(0.2351)$

$F_{par} = 1.96\text{ N}$

Q13. A student's desk has a hinged top that can open and close. She places a book on the top of the desk and then holds it in place as she opens the desk. If the book has a mass of 2.5 kg and the desk top forms an angle of 78° with the horizontal, how much force must the student exert on the book to keep it from sliding down the table? You may ignore friction in your answer.

**Answer:** First, students must calculate the weight of the book (***Fg***) using the equation *Fg* = mg. The weight of the book is therefore (2.5 kg)(9.8 m/s$^2$) = 24.5 N. The force the student must exert to keep the book from sliding down the desk is equal to the force on the book that is parallel to the surface of the desk. The magnitude of this force is calculated below:

$F_{par} = F_g \sin\Theta$

$F_{par} = (24.5\text{ N})\sin(78°)$

$F_{par} = (24.5\text{ N})(0.9781)$

$F_{par} = 24.0\text{ N}$

So, the student must exert approximately 24 N on the book to keep it from sliding off the desk.

**TI-Nspire Navigator Opportunities**

Use Navigator to capture screen shots of student progress and to retrieve the file from each student at the end of the class period. The student questions can be electronically graded and added to the student portfolio.

## Wrap Up

When students are finished with the activity, pull back the .tns file using TI-Nspire Navigator. Save grades to Portfolio. Discuss activity questions using Slide Show.

## Assessment

- Formative assessment will consist of questions embedded in the .tns file. The questions will be graded when the .tns file is retrieved by TI-Navigator™. The TI-Navigator™ Slide Show can be utilized to give students immediate feedback on their assessment.
- Summative assessment will consist of questions/problems on the chapter test, inquiry project, performance assessment, or an application/elaborate activity.