

Orthocentre Guided Investigation

Student Investigation

7 8 9 **10** 11 12



Introduction

The centre of a circle, square, rectangle or regular polygon is relatively easy to locate. The centre of a triangle on the other hand is much harder to define. In this investigation you will explore the 'orthocentre', one type of triangle centre.

The orthocentre is located at the intersection of the triangle's altitudes. When we hear the word altitude, we think of height above a surface, where the height is perpendicular to the surface. So the altitude of a triangle is a perpendicular line segment passing through a vertex of the triangle and the opposite side (or extension of the side).



The QR code or URL contains a video that demonstrates how to set up the Graphs application for this investigation. The video relates to the circumcentre, however the processes are very similar.

<https://bit.ly/Circumcentre>

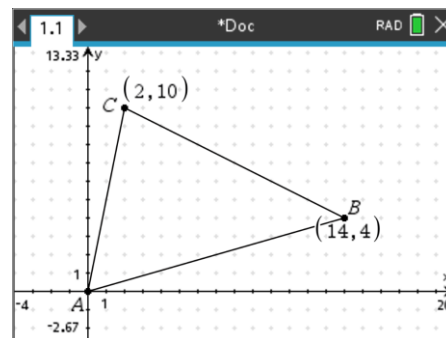
Geometry

Open a New TI-Nspire Document and insert a **Graphs Application**.

Set the window/zoom settings to Quadrant 1 and display a dot grid using the Settings option.

Draw a triangle with vertices:

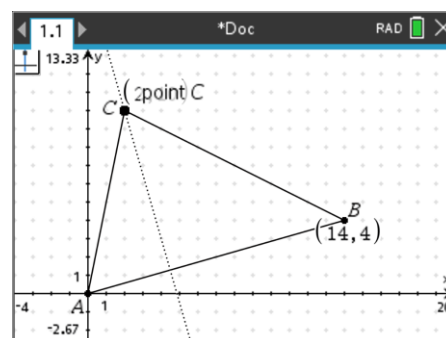
A:(0, 0) B:(14, 4) C:(2, 10)



A perpendicular line can be drawn using the construction tools:

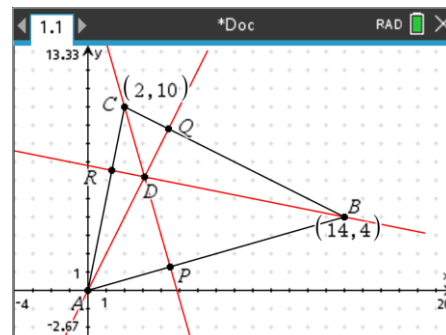
menu > **Geometry** > **Construction** > **Perpendicular**

Select side A followed by the opposite vertex (C).



Repeat this process to construct all three altitudes, then construct a point of intersection, the orthocentre!

Note: The additional points have been labelled for reference and the lines coloured for visual purposes only.



Question: 1.

Determine the gradient of side AB and hence the gradient of the altitude (PC)

Question: 2.

Determine the equation of the altitude to side AB passing through C. (Line PC)

Tip!

The translational form of a straight line is useful when the gradient and a point are known:

$y = m(x - h) + k$ is a straight line with gradient m passing through the point (h, k) .

Remember to use your calculator to check your answers.

Question: 3.

Determine the gradient of side AC and hence the gradient of the altitude (RB).

Question: 4.

Determine the equation of the altitude to side AC passing through B. (Line RB)

Question: 5.

Determine the gradient of side BC and hence the gradient of the altitude (QA)

Question: 6.

Determine the equation of the altitude to side BC passing through A. (Line QA)

Question: 7.

Use simultaneous equations to determine the point of intersection for the altitudes: QA and PC.

Extension – Ceva's Theorem

Ceva's theorem states that in order for AQ, BR and CP to be concurrent (meet at a single point D) then the following must apply:

$$\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = 1$$

Note: AP represents the distance from A to P.

Use Ceva's theorem to show that the altitudes of the triangle with vertices:

$$A:(0, 0) \quad B:(14, 4) \quad C:(2, 10)$$

are concurrent.

