Exploration 7

Teacher Notes

Exploration: Using Tables to Solve Linear Equations

Learning outcomes addressed

2.1 Solve "one-step" linear equations of the form $x + a = b$ and $ax = b$ where a and b are real numbers.

2.2 Solve "two-step" linear equations of the form $ax + b = c$ where a and b are real numbers.

Lesson Context

In organizing a catered fund-raising event, the students at Twin Oaks Junior High obtain quotes from three different locations. The quotes, in dollars, as a function of the number of guests n are:

Galaxy Inn: $f(n) = 17n$ for $n \geq 24$, but a minimum charge of \$400
Noble Pines Country Club: $g(n) = 90 + 14n$ for $n \geq 1$
Holiday Lodge: $h(n) = 200 + 12n$ for $n \geq 25$, but a minimum charge of \$500

Students are given these quotes verbally and asked to write each quote as a formula in the variable n. Then students enter each formula into the calculator application of TI-*n*spire. Once the formulas are entered, students use the spreadsheet application of TI-*n*spire to display a table of values for each function. By scrolling through these tables, students are able to determine the value of a quote corresponding to a given number of guests and, conversely, the number of guests that correspond to a particular quote. In this process, students solve informally linear equations of the form $ax + b = c$.

Lesson Launch

Have students read the telephone conversations that introduce this *Exploration*
Launch the lesson using initiating questions such as:

- How many guests are needed to exceed the minimum charge at the Galaxy Inn?
- What is the cost of 20 guests at the Holiday Lodge? answer: \$500
- What is the cost of 10 guests at the Noble Pines Country Club? answer: \$90 + \$140 or \$230

Lesson Closure

Through a class discussion, ensure that students understand how to construct a table of values for a formula (later called a "function") and how to use the table of values to solve a linear equation.

Discuss the limits of solving equations using a function table, e.g., a table lists the values of a formula for a discrete set of values of the variable so the exact solution of an equation may not be included in that set. That is, the exact solution may appear in the function table.

Student Work Sheet

Exploration: Using Tables to Solve Linear Equations

Read the telephone conversations pertaining to the three quotes given for the fund-raising school reunion at Twin Oaks Junior High.

❶. Write a formula giving the quote in dollars for n guests (where $n \geq 25$) at:

a) The Galaxy Inn $f(n) =$

b) The Noble Pines Country Club $g(n) =$

c) The Holiday Lodge $h(n) =$

❷. To access the *Define* command in the *Calculator* application, select:

(menu) > **Actions** > **Define**

Using your answers in ❶, define $f(n)$, $g(n)$ and $h(n)$.

❸. To obtain a function table for $f(x)$, access the *Lists & Spreadsheet* application by pressing (on) ▶▶▶ (enter). Then select (menu) > **Table** > **Switch to Table**.

Select the formula ***f*** by highlighting ***f*** and pressing (enter).

❹. Scroll through your function table in ❸ to find the cost of 36 guests at the Galaxy Inn. Verify your answer by returning to the calculator application and entering:

F (3 6) (enter)

Scroll through the function table to find the number of guests that would cost $1020 at the Galaxy Inn.

❺. Construct a function table for $g(x)$. Scroll through the table to find the cost of 45 guests at the Noble Pines Country Club. Verify your answer by calculating $g(45)$.

Scroll through the function table to find the number of guests that would cost $1000 at the Noble Pines Country Club.

❻. Construct a function table for $h(x)$. Scroll through the table to find the cost of 32 guests at the Holiday Lodge. Verify your answer by calculating $h(32)$.

Scroll through the function table to find the number of guests that would cost $1000 at the Holiday Lodge.

TI-nspire Investigation

Follow the instructions in the TI-*n*spire Investigation in the *Exercises*.
Then complete these statements:

a) The least expensive location for 50 guests is _____.

b) The least expensive location for 75 guests is _____.

c) The number of guests for which the Holiday Lodge and the Noble Pines offer the same price is _____.

Exploration 7: Using Tables to Solve Linear Equations

Jennifer, Steve, and their committee are planning a school reunion at Twin Oaks Junior High. They want to rent a hall and provide food and entertainment. Everyone in attendance will pay an admission fee to cover costs and the profit will be used to purchase computers for the school. To minimize the cost of the hall, Jennifer and Steve called three establishments to obtain quotes. In this *Exploration*, you will express the quotes as linear formulas and use tables to solve linear equations.

Worked Examples

Example 1

a) Write a formula that expresses the cost for n students at the Galaxy Inn (where $n \geq 24$).

b) Construct a table that displays the cost at the Galaxy for n students where $n \geq 24$. Use the table to find the cost for 36 students.

c) Use your table to determine how many students can attend for a total cost of $1020.

Solution

a) The cost for 24 students is 24 × \$17 or \$408, which is greater than the minimum charge of \$400. Therefore, if $n \geq 24$, the cost is \$17 per student. We can write this as the formula $f(n) = 17n$, where n is the number of students.

b) Before we can construct a table for $f(n)$ in TI-*n*spire, we must define it. To access the *Define* command in the *Calculator* application, we select:

(menu) > **Actions** > **Define**

Then, we enter: F (N) = 1 7 N (enter)

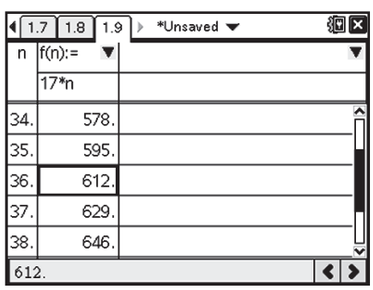
To obtain a function table for $f(n) = 17n$, we access the *Lists & Spreadsheet* application by pressing (⌂ on) ▶ ▶ ▶ (enter).

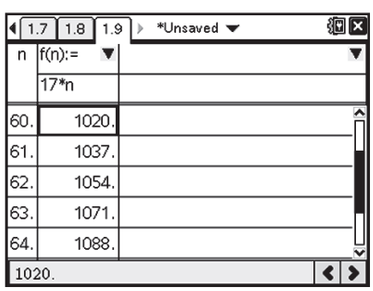
Then we select (menu) > **Table** > **Switch to Table**.

We highlight formula *f* and press ***enter*** to obtain the table. We then scroll down the *n* column to $n = 36$. Opposite $n = 36$, we see 612 (as in the display) indicating that the cost for 36 students is \$612.

c) To find the number of students who can attend for \$1020, we scroll down the table until we see 1020 in the column marked **f(n):**. As shown in the display, this amount appears opposite $n = 60$. That is, 60 students can attend for \$1020.

Note: This function table is valid only for $n \geq 24$. In the TI-*n*spire *Investigation* in the *Exercises* you will learn how to start the table at a particular value of n.





Example 2

a) Write a formula that expresses the cost for n students at the Noble Pines Country Club.

b) Construct a table that displays the cost at the Noble Pines Country Club for n students. Use the table to find the cost for 45 students.

c) Use your table to determine how many students can attend for a total cost of \$1000.

Solution

a) The cost is \$90 plus \$14 per student. We can write this as the formula $g(n) = 14n + 90$, where n is the number of students.

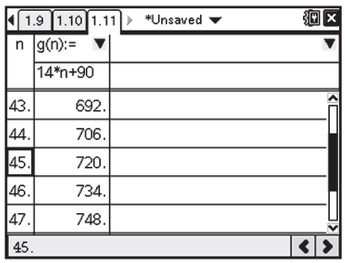
b) Before we can construct a table for $g(n)$ in TI-*n*spire, we must define it. To access the *Define* command in the *Calculator* application, we select:

(menu) > **Actions** > **Define**

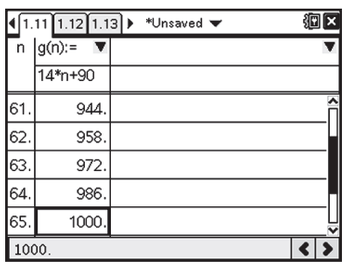
Then, we enter: G (N) = 1 4 N + 9 0 (enter)

To obtain a function table for $g(n) = 14n + 90$, we access the *Lists & Spreadsheet* application by pressing (on) ▶ ▶ ▶ (enter).

Then we select (menu) > **Table** > **Switch to Table**.



We highlight formula g and press (enter) to obtain the table. We then scroll down the n column to $n = 45$. Opposite $n = 45$, we see 720 (as in the display) indicating that the cost for 45 students at the Noble Pines Country Club is \$720.



c) To find the number of students who can attend for \$1000, we scroll down the table until we see 1000 in the column marked **g(n):**. As shown in the display, this amount appears opposite $n = 65$. That is, 65 students can attend for \$1000.

Example 3

a) Write a formula that expresses the cost for n students at the Holiday Lodge (where $n \geq 25$).

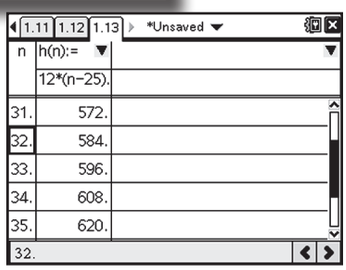
b) Construct a table that displays the cost at the Holiday Lodge for n students (where $n \geq 25$). Use the table to find the cost for 32 students.

c) Use your table to determine how many students can attend for a total cost of \$1000.

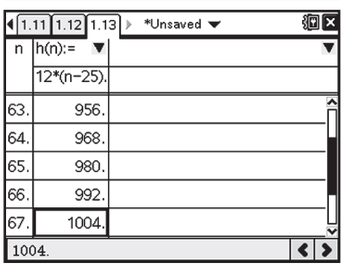
Solution

a) The cost is \$500 plus \$12 per student, for those in excess of 25. We write this as the formula $h(n) = 12(n - 25) + 500$, where n is the number of students ($n \geq 25$).

b) Before we can construct a table for $g(n)$ in TI-*n*spire, we define it as in the previous examples using $h(n) = 12(n - 25) + 500$. We then construct the table for $h(n)$ as shown in the previous examples.



When we scroll down column n in the table to $n = 32$, we see that the number opposite 32 is 584. That is, the cost of 32 guests at Holiday Lodge is \$584.



c) To determine the number of students who could attend for \$1000, we cursor down the column with heading **h(n):=** until we reach the number 1004. This is the number opposite $n = 67$, i.e. 67 students would cost \$1004. Therefore, 66 students could attend for \$1000 and there would be \$8 left over.

Exercises and Investigations

❶. a) Define each of these formulas:
i) $f1(x) = 5x + 8$ ii) $f2(x) = 7x - 3$ iii) $f3(x) = -x/2 + 4$
b) Construct a function table for each formula.
Use the function tables to evaluate the following:
i) $f1(4)$ ii) $f2(6)$ iii) $f3(^{-}6)$
c) Use the function tables to find the value of x such that:
i) $f1(x) = 18$ ii) $f2(x) = ^{-}24$ iii) $f3(x) = 0$

❷. a) Construct a flow chart for formulas $f1(x)$, $f2(x)$, and $f3(x)$ in *Exercise 1a* above. Then construct the "undo" flow chart to find the inverse formulas $f1^{-1}(x)$, $f2^{-1}(x)$, and $f3^{-1}(x)$.

b) Calculate $f1^{-1}(18)$, $f2^{-1}(^{-}24)$, and $f3^{-1}(0)$.

c) Compare your answers in *part b* of this exercise with your answers in *Exercise 1c*). Describe what you discover.

❸. a) Follow the steps in *Example 1* on page 38 to construct a function table for the costs of n guests at the Galaxy Inn (when $n \geq 25$).

b) Use your table to find the number of students who can attend for a total cost of $1207.

c) If Jennifer's budget is $1400, what is the greatest number of students who can attend if they choose the Galaxy Inn?

❹. a) Construct tables that display the costs at the Galaxy Inn, the Noble Pines Country Club and the Holiday Lodge for n students where $n \geq 25$.

b) Which facility is the least expensive for 50 students?

c) Which facility is the least expensive for 75 students?

d) Which facility will accommodate the most students at a cost of $900?

❺. The formula for the n^{th} odd integer is $z(n) = 2n - 1$.

a) Define $z(n)$ in the calculator application of TI-*n*spire.

b) Use $z(n)$ to determine the 21st odd integer.

c) Construct a function table for $z(n)$. Use your function table to find the values of n for which $z(n) = 35$, for which $z(n) = 47$ and for which $z(n) = 81$.

❻. a) Construct the flow chart that defines the formula for $z(n)$ where $z(n) = 2n - 1$. Then construct the "undo" flow chart to find the inverse formula $z1(n)$.

b) Evaluate $z1(35)$, $z1(47)$, and $z1(81)$.

c) Compare the answers in *part b* of this exercise with your answers in *Exercise 5c*). Describe what you discover. Why might you expect this?

Some Non-linear Formulas

❼. a) The formula for the n^{th} perfect square is $s(n) = n^2$.

a) Define $s(n)$ in the calculator application of TI-*n*spire.

b) Use $s(n)$ to determine the 18th perfect square.

c) Construct a function table for $s(n)$. Use your function table to find the values of n for which $s(n) = 196$ and for which $s(n) = 289$.

d) Use your function table to find $\sqrt{529}$.

❽. a) The formula for the n^{th} triangular number is $t(n)$ where $t(n) = n(n + 1)/2$.

a) Define $t(n)$ in the calculator application of TI-*n*spire.

b) Evaluate $t(3)$, $t(7)$, and $t(13)$.

c) Construct a function table for $t(n)$. Use your function table to find the value of n for which $t(n) = 171$.

TI-nspire Investigation

Define the following cost formulas for n guests at each location:
Galaxy Inn cost formula: $g(n) = 17n$
Holiday Lodge cost formula: $h(n) = 200 + 12n$
Noble Pines cost formula: $j(n) = 90 + 14n$

Note
▸ denotes the right-arrow key on the NavPad

To create a function table for $g(n)$, press (on) ▸ ▸ ▸ (enter) to access *Lists & Spreadsheet*

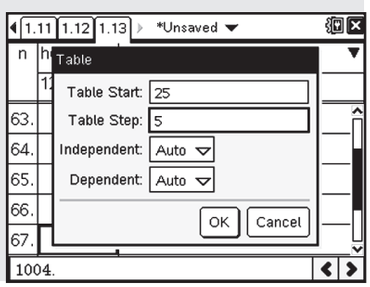
Select (menu) > **Table** > **Switch to Table**.
Then highlight formula g and press (enter).

Then press ▸ and select formula h from the template.

Press ▸ again and select formula j from the template.

To display these function tables for $n \geq 25$ in steps of 5 press:
(menu) > **Table** > **Edit Table Settings**.

Complete the template as shown in the display. Tab down to (OK) and press (enter).
Use your table to find the least expensive locations for 50 guests and for 75 guests.
Compare your answers with those you obtained in *Exercise 4b* and *4c* above.



Answers to the Exercises & Hints for the Investigations

Exploration 7

❶. a)

Define $f1(x)=5\cdot x+8$	Done
Define $f2(x)=7\cdot x-3$	Done
Define $f3(x)=\frac{-x}{2}+4$	Done

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❶. b) i)

	f1(x):= 5*x+8
0.	8.
1.	13.
2.	18.
3.	23.
4.	28.

ii)

	f2(x):= 7*x-3
2.	11.
3.	18.
4.	25.
5.	32.
6.	39.

iii)

	f3(x):= -x/2+4
-6.	7.
-5.	6.5
-4.	6.
-3.	5.5
-2.	5.

❶. c) i)

	f1(x):= 5*x+8
0.	8.
1.	13.
2.	18.
3.	23.
4.	28.

ii)

	f2(x):= 7*x-3
-6.	-45.
-5.	-38.
-4.	-31.
-3.	-24.
-2.	-17.

iii)

	f3(x):= -x/2+4
4.	2.
5.	1.5
6.	1.
7.	0.5
8.	0.

❷. a) $f1^{-1}(x) = (x - 8)/5$; $f2^{-1}(x) = (x + 3)/7$; $f3^{-1}(x) = 2(x - 4)$

b) $f1^{-1}(18) = 2$; $f2^{-1}(-24) = -3$; $f3^{-1}(0) = -8$

c) The answers in *part b* are the same as the answers in *Exercise 1c*, because $f^{-1}(x)$ is the "undo" or inverse function that undoes what $f(x)$ does. That is, $f^{-1}(f(x)) = x$.

❸.

	x	f(x):= 17*x
	67.	1139.
	68.	1156.
	69.	1173.
	70.	1190.
	71.	1207.

71.

	x	f(x):= 17*x
	79.	1343.
	80.	1360.
	81.	1377.
	82.	1394.
	83.	1411.

1394.

b) The table shows that 71 students can attend at the Galaxy Inn for $1207.

c) The table shows that 82 students would cost $1394 and 83 students would cost $1411, so 82 is the maximum number of students who could attend the Galaxy Inn.

❹. a)

	f(n):= 17*n
46.	782.
47.	799.
48.	816.
49.	833.
50.	850.

	f(x):= 90+14*x
46.	734.
47.	748.
48.	762.
49.	776.
50.	790.

	f(n):= 200+12*n
46.	752.
47.	764.
48.	776.
49.	788.
50.	800.

b) Comparing the tables for $n = 50$ reveals that the Noble Pines is the least expensive option for 50 guests.

c) The Holiday Lodge is the least expensive option for 75 guests.

d) For $900, the Holiday Lodge will accommodate 58 guests.

❺. a) & b)

Define $z(n)=2\cdot n-1$	Done
$z(21)$	41

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❺. c)

	z(n):= 2*n-1
14.	27.
15.	29.
16.	31.
17.	33.
18.	35.

	z(n):= 2*n-1
20.	39.
21.	41.
22.	43.
23.	45.
24.	47.

	z(n):= 2*n-1
37.	73.
38.	75.
39.	77.
40.	79.
41.	81.

Exploration 7 cont'd

❻. a) & b)

Define $z1(n)=\frac{n+1}{2}$	Done
$z1(35)$	18
$z1(47)$	24
$z1(81)$	41

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c) The answers in *part b* are the same as the answers in *Exercise 5c*, because $z1(n)$ is the "undo" or inverse function that undoes what $z(n)$ does. That is, $z1(z(n)) = n$.

❼. a) & b)

Define $s(n)=n^2$	Done
$s(18)$	324

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❼. c) & d)

	s(n):= n^2
10.	100.
11.	121.
12.	144.
13.	169.
14.	196.

	s(n):= n^2
15.	225.
16.	256.
17.	289.
18.	324.
19.	361.

	s(n):= n^2
22.	484.
23.	529.
24.	576.
25.	625.
26.	676.

❽. a) & b)

Define $t(n)=\frac{n\cdot(n+1)}{2}$	Done
$t(3)$	6
$t(7)$	28
$t(13)$	91

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❽. c)

	t(n):= n*(n+1)/2
14.	105.
15.	120.
16.	136.
17.	153.
18.	171.

18.

Note: When defining the product $n(n - 1)$, it is important to remember to insert the × symbol between the n and the $(n - 1)$. It will show as a dot. If you omit this, the calculator will give you an error message.

Hint for the TI-nspire Investigation

The least expensive locations for 50 and 75 guests are displayed in the highlighted cells of the spreadsheet.

n	g(n):= 17*n	h(n):= 200+12*n	j(n):= 90+14*n
35	595.	620.	580.
40	680.	680.	650.
45	765.	740.	720.
50	850.	800.	790.
55	935.	860.	860.

790.

n	g(n):= 17*n	h(n):= 200+12*n	j(n):= 90+14*n
60	1020.	920.	930.
65	1105.	980.	1000.
70	1190.	1040.	1070.
75	1275.	1100.	1140.
80	1360.	1160.	1210.

1100.