Count the Differences

About the Lesson

In this activity, students will connect how they found slope for linear functions to finite differences for polynomials to find the degree of a polynomial. As a result, students will:

- Find the degree of polynomials using finite differences.

Vocabulary

- degree
- polynomial function
- matrices (Extension)
- systems of equations (Extension)

Teacher Preparation and Notes

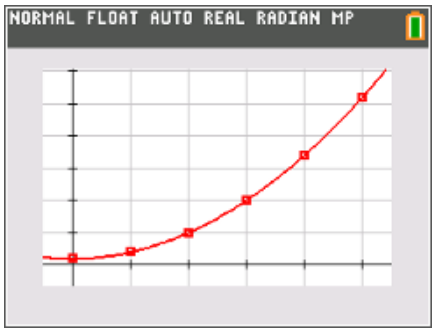
- Problem 1 is meant to be a teacher demonstration to review finding slope and connecting the process to finite differences.
- Teachers can lead Problems 2 and 3 or let students work on their own. Students are asked to find an equation for a quadratic and cubic equation. If students have gone over a process for how to find an equation, then they can solve for them algebraically. If not, then students can use guess and check and use graphing to check their answer.
- Extension uses inverse matrices to solve a system of equations. (Students will need to know how to use matrices on their graphing calculator.)

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

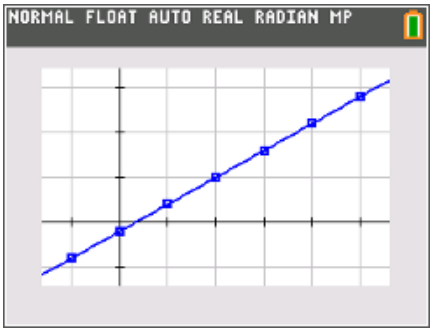
- Count_the_Differences_Student.pdf
- Count_the_Differences_Student.doc
- L1.8xl
- L2.8xl
- L3.8xl
- L4.8xl
- L5.8xl
- L6.8xl

Tech Tip: Before beginning the activity, the files L1.8xl through L6.8xl need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE Software.

Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing 2nd zoom [format] to change the ALPHA PRGM h settings. If your students are using TI-84 Plus, they could use GridDot.

Problem 1 – Review Linear Functions

In this problem, students will use the data lists **L1** and **L2** to find the 1st differences. This will be the slope of data given and will show as a linear function in a graph. The teacher should make the connection for students that the first finite differences are the same and the linear function is a first-degree polynomial.



Discussion Questions:

- *What is the change (difference) between consecutive y-values?*
- *What degree is the function that models the data?*

1. Use the table to help you record the differences.

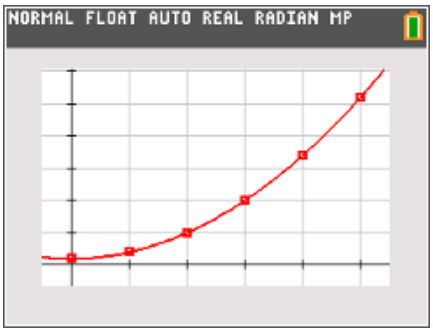
Answers:

x **(L1)**	***y*** **(L2)**	**1st difference**
–1	–4	3
0	–1	3
1	2	3
2	5	3
3	8	3
4	11	3
5	14	—

2. What is the relationship between the degree of a linear function and the number of times you had to subtract the *y*-values to find the same difference?

Answer: A linear function is first degree and the first differences were the same.

Problem 2 – Finding Finite Differences, Part 1



In this problem, students will find the differences between the *y*-values given in table. Students should have to find the difference twice to find the same difference between consecutive terms. Teachers should make sure students are subtracting the correct terms. Students should try to find the function any way they can.

Discussion Questions:

- *How many times did you have to find the difference?*
- *What is the relationship between the degree of the polynomial that models the data and the number of times you need to find the difference between the term values?*

3. Use the table to help you record the differences.

Sample Answers:

x (L3)	*y* (L4)	1st difference	2nd difference	3rd difference
0	1	1	2	0
1	2	3	2	0
2	5	5	2	0
3	10	7	2	—
4	17	9	—	—
5	26	—	—	—

4. How many times did you have to find the difference between the *y*-values until they were all the same?

Answer: 2

5. What is the function that models the data?

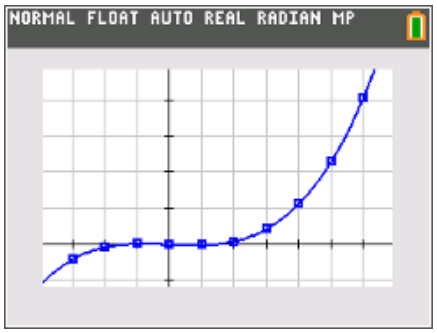
Answer: $f(x) = x^2 + 1$

6. What type of function models the data?

Answer: quadratic or function with degree 2

Problem 3 – Finding Finite Differences, Part 2

This problem is the same as Problem 2, but students will have to find the difference three times and determine a cubic polynomial.



7. Use the table to help you record the differences.

Sample Answers:

x	*y*	1st difference	2nd difference	3rd difference
–3	–21	17	–12	6
–2	–4	5	–6	6
–1	1	–1	0	6
0	0	–1	6	6
1	–1	5	12	6
2	4	17	18	6
3	21	35	24	6
4	56	59	30	—
5	115	89	—	—
6	204	—	—	—

8. How many times did you have to find the difference between the *y*-values until they were all the same?

Answer: 3

9. What is a function that models the data?

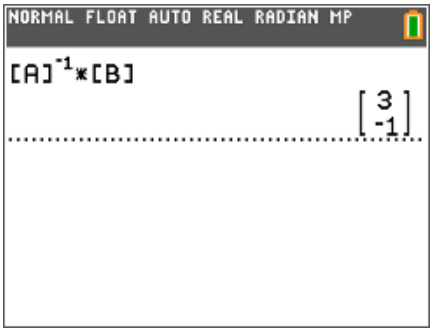
Answer: $f(x) = x^3 - 2x$

10. What type of function models the data?

Answer: cubic or function with degree 3

Extension – Using Matrices to Find Equations

In the extension, students will write a system of equations and solve the system using matrices to find the polynomial that fits the data.



11. Write the general form of a linear equation two times. In one of them, substitute the *x*- and *y*-values of third point. In the other, substitute the *x*- and *y*-values of the fourth point. You will now have a system of equations.

Answer: $5 = 2a + b$
$2 = a + b$

12. Now write the equation as a matrix equation. (Part of it is done for you.)

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Answer: $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

13. Use the inverse of a matrix to solve the matrix equation. You should find the same values for *a* and *b* as in Problem 1.

Answer: $\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}5\\ 2\end{bmatrix}$

$$\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}3\\ -1\end{bmatrix}$$

14. You can use the same process for Problems 2 and 3 by using the general equations for a quadratic and cubic polynomial and substituting in the correct number of points to get the equation. Write the system of equations and the matrix equation below before solving the matrix equation on your calculator.

Sample Answers:

Problem 2: $1=c$

$2=a+b+c$

$5=4a+2b+c$

$$\begin{bmatrix}0 & 0 & 1\\ 1 & 1 & 1\\ 4 & 2 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 5\end{bmatrix}$$

Problem 3: $0=d$

$-1=a+b+c+d$

$4=8a+4b+2c+d$

$21=27a+9b+3c+d$

$$\begin{bmatrix}27 & 9 & 3 & 1\\ 8 & 4 & 2 & 1\\ 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}=\begin{bmatrix}21\\ 4\\ -1\\ 0\end{bmatrix}$$