Energy of a Roller Coaster

SCIENCE NSPIRED

Science Objectives
- Students will explore the relationship between work and energy.
- Students will explore energy transformations related to the motion of a roller coaster with and without friction.

Vocabulary
- conservative vs. non-conservative forces
- gravitational potential energy
- kinetic energy
- total mechanical energy
- work

About the Lesson
- This lesson begins with an exploration of the relationships between work, potential energy, and kinetic energy using. The simulation uses a block that is either lifted and dropped or is pulled up an incline and allowed to slide back down. These concepts are then applied to a simulation of a roller coaster car moving along a track.
- As a result, students will:
    - Observe that an external force doing positive work increases the mechanical energy of an object, while friction causes a decrease in mechanical energy.
    - Describe how energy transforms between potential and kinetic forms, and, in the absence of friction, the total mechanical energy remains constant.
    - Relate the concepts of work and energy to the design and motion of a roller coaster.
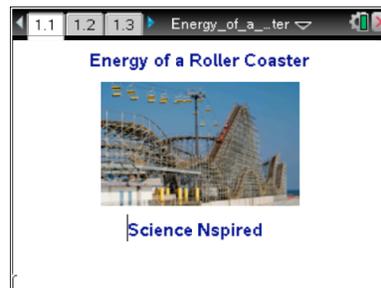
TI-Nspire™ Navigator™
- Send out the *Energy_of_a_Roller_Coaster.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials
- Compatible TI Technologies: ⬛ TI- Nspire™ CX Handhelds, 📱 TI-Nspire™ Apps for iPad®, 💻 TI-Nspire™ Software

Tech Tips:
- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
- Energy_of_a_Roller_ Coaster_Student.doc
- Energy_of_a_Roller_ Coaster_Student.pdf

TI-Nspire document
- Energy_of_a_Roller_ Coaster.tns

Discussion Points and Possible Answers

Allow students to read the background information on the student activity sheet.

This file is divided into four problems. The first problem simulates a block that students lift and drop. Students calculate the work done and compare it to the energy acquired by the block. The second problem explores these same ideas with a block on an incline. The third problem introduces friction between the block and the incline. The fourth problem simulates a roller coaster car moving along a track with and without friction. Students should follow the handout and answer the questions as they manipulate the simulations. A few summary questions are included in the .tns file.

Move to pages 1.2 and 1.3.

1. Have students read the instructions and background materials on page 1.2 before moving on to the simulation. After reading the directions on page 1.3, select ✕ to close the pop-up box.

2. Students should grab the block by the point in the middle and lift it above the surface. The instructions and questions in the student handout guide them through the exploration.
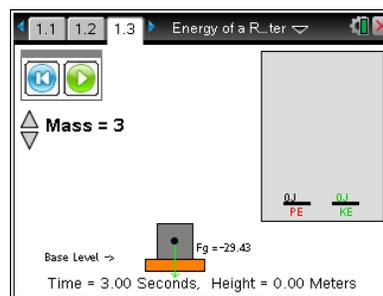
> **Tech Tip:** When students grab the block, the red arrow showing applied force may remain after they release the block. They can tap the block once to make the arrow disappear.

Students should answer questions 1–3 on their activity sheets as they work through the simulation.

Q1. Grab the block and lift it above the surface. Then use the spaces below to record the lifting force and the height to which you lift the block. Use these numbers to calculate the work you do on the block. Observe what happens to the gravitational potential energy as you lift the block. Select the play button, and observe the gravitational potential and kinetic energies of the block as it falls.

Sample Answers: A. $F = $ **29.43** N, B. $h = $ **1.23** m, C. $W = F \cdot h = $ **36.2** J

Q2. Describe what happens to the gravitational potential energy of the block as you lift it. Compare the potential energy at the highest point to the work done in lifting the block.

Sample Answer: The potential energy of the block increases as it is lifted. The amount of potential energy gained is equal to the amount of work done on the block.

Q3. Describe what happens with the gravitational potential and kinetic energies as the block falls. What do you observe about the sum of the two energies as the block falls? Compare the kinetic energy of the block when it reaches the surface to the potential energy when you released it.

Sample Answer: The kinetic energy increases as the potential energy decreases during the fall. The sum of the two energies remains constant. At the surface, all of the initial potential energy has been transformed into kinetic energy.

Move to pages 2.1–2.2.

3. Read the information on pages 2.1 and 2.2. After reading the directions on page 2.3, select ☒ to close the pop-up box.

Move to page 2.3.

4. On this page students should grab the point in the middle of the block near the corner of the incline and move it up to pull the 3-kg block up the incline. The incline is frictionless. Students should observe that the gain in gravitational potential energy is still equal to the work done in pulling the block to the top. The applied force is equal to the component of the block's weight, which is parallel to the incline.
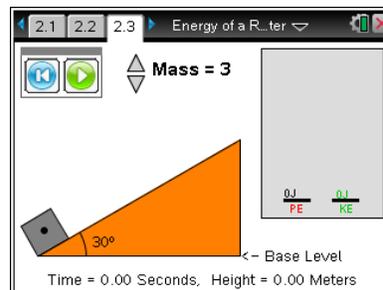
Students should answer questions 4-6 below before moving to page 2.4.

Q4. Drag the block to the top of the incline, and record the distance and the force required to pull the block to this location. The work done on the block is the product of these two numbers. Observe the gravitational potential energy as the block is pulled up the hill. Select the play button, and observe the potential and kinetic energies of the block as it slides down the incline.

   Sample Answer: A. F = **20** N, B. d = **8.42** m, C. $W = F \cdot d$ = **168.4** J

Q5. The block has a mass of 3 kg, and the hill is a 30° incline. Demonstrate how the height of the block at the top of the hill can be calculated. Use this height to calculate the gravitational potential energy at the top of the hill. Compare your calculation of potential energy to the work calculated in the previous question.

   Sample Answer: $h = d \cdot \sin \theta$ = 8.42 m $\cdot$ sin 30° = 4.21 m

   $PE = mgh$ = 3 kg $\cdot$ 9.8 N $\cdot$ 4.21 m = 123.774 J. This is the same as the work done on the block.

Q6. Describe what happens with the potential and kinetic energies as the block slides down the hill. What do you observe about the sum of the two energies as the block slides? Compare the kinetic energy of the block when it reaches the surface to the gravitational potential energy when you released it.

   Sample Answer: The kinetic energy increases as the potential energy decreases when the block

slides down the incline. The sum of the two energies remains constant. All of the initial potential energy is transformed into kinetic energy.

Move to pages 2.4–2.6.

Have students answer the questions on the device, the activity sheet, or both.

Q7.   The gravitational potential energy of an ideal roller coaster at its highest point is equal to the work required to get it to the top of the hill.

   Answer: A. True

Q8.   Choose the correct statement(s) about kinetic energy (KE) and potential energy (PE) based on your observations of the block on the frictionless incline. (There is more than one correct statement.)

   Answers:
   B. As the block slides down the hill, its KE increases at the same rate as its PE decreases.
   C. As the block slides, the sum of its KE and PE remains constant.
   D. The greater the height of the block when it is released, the more KE it will have when it reaches the bottom.

Q9.   On an actual roller coaster, how does it gain its initial gravitational potential energy?

   Sample Answer: A chain pulls the roller coaster to the top of the first hill and does work on it.

Move to pages 3.1 and 3.2.

5.   Students should read the information on page 3.1. After reading the directions on page 3.2, select ☒ to close the pop-up box.

6.   Select the play button on page 3.2, and observe the work and energy bar graphs as the block slides.
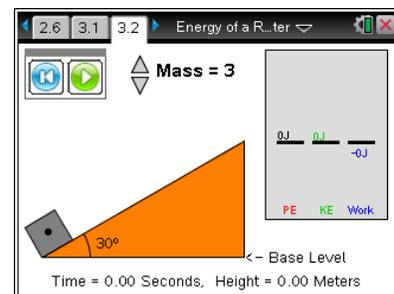
This simulation includes a friction force acting on a 3-kg block as the block slides down a 30° incline. The friction force does negative work on the block as it slides down the incline. Students should observe that this decreases the mechanical energy of the block.

Students should answer questions 10 and 11 on the activity sheet before moving to page 4.1.

Q10. Note the potential energy of the block at the top of the hill. Select the play button, and observe the work and energy bar graphs as the block slides. Describe the changes you observe in the potential energy and the kinetic energy of the block as it slides. Compare the kinetic energy at the bottom of the hill to the potential energy at the top.

   Sample Answer: The potential energy decreases and the kinetic energy increases as the block slides down the incline, but the kinetic energy of the block at the bottom is less than the potential energy of the block at the top. Some energy is missing.

Q11. How much work was done by the friction force? What effect did this appear to have on the energy of the block?

   Sample Answer: The amount of work done by friction is equal to the missing energy. Friction appears to take energy away from the block.

Move to pages 4.1 and 4.2.

Page 4.1 simulates a roller coaster car moving along a track. The height and velocity of the car are displayed. On the right side of the screen are bar graphs for potential energy, kinetic energy, total mechanical energy, and work done by friction. The total mechanical energy is the sum of the potential and kinetic energies.
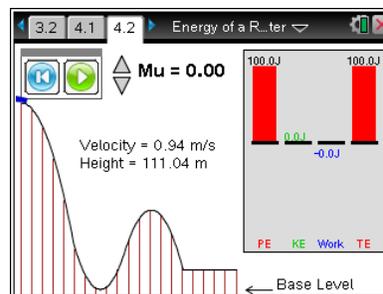
7. After reading the directions students select [X] to close the pop-up box. The roller coaster car is already at the top of the track. Students can select the play button [▶] will release the car. Selecting the reset button [⏮] will place the car back at the top of the roller coaster. They may change the friction, μ, by selecting the up and down arrows (▲ or ▼) for Mu.
   Students should note the increase in velocity as the coaster moves up and down hills. A graph shows the corresponding changes in potential, kinetic, and total energy.

Students should answer questions 12-21 on the activity sheet before moving to page 4.3.

Q12. Set the value of Mu to zero. Select the down arrow ▼ a couple times after it shows Mu = 0.00. Start the car and allow it to move through the track. Observe the bar graphs as the car moves along the track. Describe the changes you observe in the potential energy, kinetic energy, and total energy. Comment on how the three energy measurements appear to relate to each other.

Sample Answer: The potential energy decreases as the car moves down a hill and increases as it moves up a hill. The kinetic energy increases as the car moves down a hill and decreases as it moves up a hill. The total energy stays constant. The energy transforms back and forth between potential and kinetic.

Q13. Reset the car and set the value of Mu to zero. Stop the car at the bottom of the first hill. Record the values of the PE, KE, TE, height (h), and velocity (v). Repeat these steps, this time stopping at the top of the second hill. Repeat the steps again, this time stopping at the bottom of the second hill.

Sample Answers:

top of the 1st hill	PE = _100_ J	KE = ___0_ J	TE __100_ J	h = _111_ m	v = __0.94_ m/s
bottom of 1st hill	PE = ___0_ J	KE = _100_ J	TE __100_ J	h = ___0_ m	v = _46.68_ m/s
top of the 2nd hill	PE = _42.7_ J	KE = _57.3_ J	TE __100_ J	h = _47.4_ m	v = _35.23_ m/s
bottom of 2nd hill	PE = _10.8_ J	KE = _89.2_ J	TE __100_ J	h = 12.03_ m	v = _44.08_ m/s

Q14. Calculate the ratio of the gravitational potential energy at the top of the first hill to the gravitational potential energy at the top of the second hill. Now calculate the ratio of the height of the first hill to the height of the second hill. What do you observe about the two ratios? What does this tell you about the relationship between potential energy and height?

Sample Answers: $\frac{PE_1}{PE_2} = \frac{100}{42.7} = 2.34$; $\frac{h_1}{h_2} = \frac{111}{47.4} = 2.34$

The two ratios are the same. The potential energy of the car is directly proportional to the height of the track.

Q15. Calculate the ratio of the kinetic energy at the bottom of the first hill to the kinetic energy at the top of the second hill. Now calculate the ratio of the speed at the bottom of the first hill to the speed at the top of the second hill. What do you observe when you compare these two ratios? What can you infer about the relationship between the kinetic energy of the roller coaster and its speed?

Sample Answers: $\frac{KE_1}{KE_2} = \frac{100}{89.2} = 1.12$; $\frac{v_1}{v_2} = \frac{46.68}{44.08} = 1.05$

The two ratios are not the same. The ratio of the kinetic energies is larger than the ratio of the speeds. This implies that kinetic energy and speed are not directly proportional. The kinetic energy is actually proportional to the square of the speed. As a result, doubling the kinetic energy of the car does not double its speed.

Q16. At the bottom of the second hill, what percentage of the roller coaster's total energy is in the form of potential energy? What percentage is in the form of kinetic energy? What percentage of the maximum speed of the roller coaster is its speed at the bottom of the second hill?

Sample Answers: $\frac{PE}{TE} = \frac{10.8}{100} = 10.8\%$; $\frac{KE}{TE} = \frac{89.2}{100} = 89.2\%$; $\frac{v_2}{v_1} = \frac{44.08}{46.68} = 94\%$

Q17. Move the roller coaster back to the beginning of the track. Change the coefficient of friction to μ = 0.1 so that friction will act on the roller coaster and do negative work as it moves along the track. Select the play button and observe the bar graphs as the car moves along the track. A new bar graph measures the work done by the friction force. What happens to the total mechanical energy of the car as it moves along the track?

Sample Answer: The total mechanical energy of the roller coaster decreases steadily.

Q18. You have probably heard a statement similar to "Energy is neither created nor destroyed, but it may change forms." With this in mind, how can you explain what is happening to the total mechanical energy of the car? What effect is friction having on the car? (Hint: Think about what happens when you rub your hands together rapidly.)

Answer: As friction does negative work, the mechanical energy decreases. The work done by friction transforms mechanical energy into heat, a non-mechanical form of energy. As a result, the temperature of the track and the wheels increases.

Q19. Friction and air resistance are classified as non-conservative forces. In contrast, the force of gravity is classified as a conservative force. Explain what this means in the context of this simulation and your observations above. (Hint: In physics, a quantity is conserved if it remains constant as other quantities change.)

Sample Answer: When gravity does work on an object, its total mechanical energy is not changed even though energy is transformed between potential and kinetic. This means mechanical energy is conserved. When friction and air resistance do work, however, the mechanical energy of an object decreases as it is transformed into thermal energy. As a result, mechanical energy is not conserved.

Q20. Compare the speed of the car at the bottom of the first hill in this simulation to the speed of the car at the bottom of the first hill when there was no friction. What difference do you observe? Is this consistent with your observations above?

Sample Answer: The speed is 30.4 m/s with friction and 34.3 m/s without friction. The speed is less when friction is present. This means that not all of the initial gravitational potential energy has been transformed into kinetic energy. Some has been turned into thermal energy.

Q21. With friction opposing the motion, could a roller coaster make it to the top of a hill, which has the same height as the starting hill? Explain your answer.

Sample Answer: A roller coaster could not get back to its starting height. Since work done by friction decreases mechanical energy, the roller coaster would stop at a lower height. Designers must make subsequent hills smaller than the first hill.

Move to pages 4.3–4.6.

Have students answer questions 22-25 on the device, the activity sheet, or both.

Q22. A roller coaster achieves maximum speed (ignore friction) _____.

Answer: B. at the lowest position of the track

Q23. To increase the maximum speed of a roller coaster, a designer should _____.

Answer: D. make the first hill taller

Q24. In the absence of friction, the greatest height a roller coaster can achieve _____.

    Answer: C. is equal to the height of the first hill

Q25. A roller coaster will stop _____.

    Answer: A. when its potential energy is equal to its total energy

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TI-Nspire Navigator Opportunities

Take screenshots of the entire class to show the car along various parts of the track. Discuss with students the variation in the velocity and height. Throughout the activity, discuss the simulations with students using Slide Show. At the end of the lab, collect the .tns files and save to Portfolio.

Wrap Up

When students are finished with the activity, retrieve the .tns file using TI-Nspire Navigator and collect the student handouts. Save grades to Portfolio. Discuss activity questions using Slide Show.

Assessment

- Formative assessment will consist of questions embedded in the .tns file and student responses on the handout. The questions will be graded when the .tns file is retrieved. The Slide Show will be utilized to give students immediate feedback on their assessment.
- Summative assessment will consist of questions/problems on the chapter test.