Math Objectives

- Students will determine the formula for the volume of a cylinder.
- Students will relate the volume of a cylinder to the volume of a cone in order to determine the formula for the volume of a cone.
- Students will relate the volume of a cylinder to the volume of a sphere to determine the formula for the volume of a sphere.
- Students will calculate the volume of a cylinder, cone, and sphere with given dimensions.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- cylinder
- sphere
- radius
- volume
- cone
- height
- diameter

About the Lesson

- This lesson involves the volume formula for cylinders, cones, and spheres. As a result, students will:
    - Change the height and the radius of a cylinder to see the effects on the volume.
    - Write the formula for the volume of a cylinder (i.e. $V = \pi r^2 h$).
    - Discover the formula for the volume of a cone using their knowledge of the formula of the volume of a cylinder.
    - Discover the formula for the volume of a sphere using their knowledge of the formula of the volume of a cylinder.
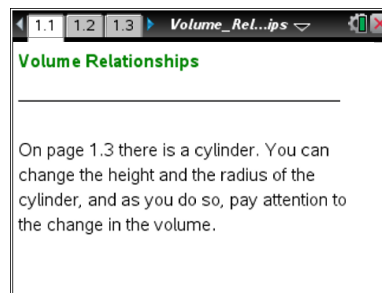
TI-Nspire™ Navigator™

- Send a document.
- Use Live Presenter to demonstrate and provide a means for students to share their thinking.
- Use Quick Poll to assess students' understanding.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Materials:
Student Activity
Volume_Relationships_Student.pdf
Volume_Relationships_Student.doc

TI-Nspire document
Volume_Relationships.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty with dragging the point, check to make sure that the cursor becomes hand (☝). Then be sure that they press ctrl 🖐 to close the hand (✊).
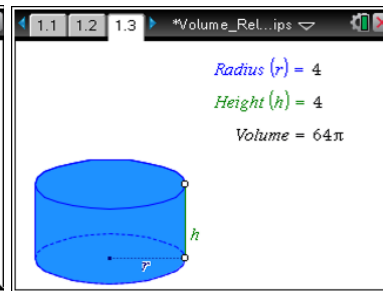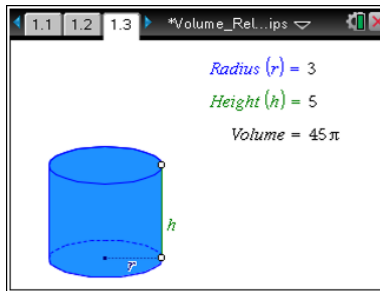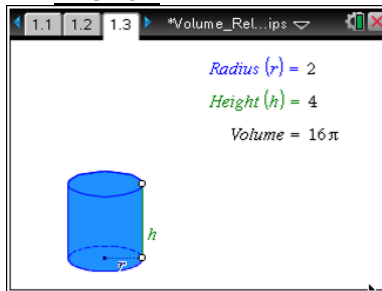
Move to page 1.3.

Move the cursor to a point near an open circle, and press ctrl 🖐. You will see the closed hand ✊. Use the NavPad to change the radius or the height.
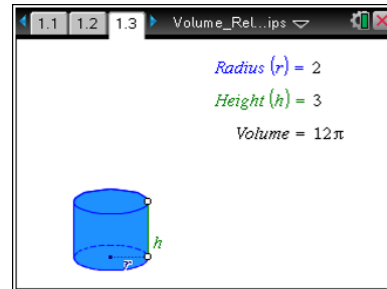
1. On Page 1.3, a cylinder is shown.
    a. The first row of the table below has been completed for you. Complete the rest of the table by:
        - changing the radius and height,
        - recording the corresponding volume, and
        - completing the calculation.

Answer:

Radius	Height	Volume	Volume ÷ (Height x π)
2	3	12π	$\dfrac{12\pi}{3\pi} = 4$
2	4	16π	$\dfrac{16\pi}{4\pi} = 4$
2	5	20π	$\dfrac{20\pi}{5\pi} = 4$
3	3	27π	$\dfrac{27\pi}{3\pi} = 9$

Radius	Height	Volume	Volume $\div$ (Height x π)
3	4	36π	$\dfrac{36\pi}{4\pi} = 9$
3	5	45π	$\dfrac{45\pi}{5\pi} = 9$
4	3	48π	$\dfrac{48\pi}{3\pi} = 16$
4	4	64π	$\dfrac{64\pi}{4\pi} = 16$
4	5	80π	$\dfrac{80\pi}{5\pi} = 16$

b. Look for a pattern in the table from part a. Using your findings, predict what you think the volume of a cylinder is with a radius of 5 and a height of 3. Explain your reasoning.

Answer: 75π. You square the radius and multiply by the height and by π.

c. Change the radius to 5 and the height to 3. How does your prediction for the volume of this cylinder compare to the actual volume of the cylinder? If your value is different, explain why you think your value is different.

Answer: Depending on their answers to part 2b, students will respond that it is the same or different based on what they see on the calculator.

d. Look back at the table, and review your findings. Based on your data, predict the volume of a cylinder with radius 5 and height 4. Record your prediction. Check your prediction by changing the dimensions. Was your prediction correct? If not, explain why your prediction is different than the correct value.

Answer: 100π. Depending on their guess, students will respond yes or no based on what they see on the calculator.

e. Use your pattern to write the formula for the volume of a cylinder with radius *r* and height *h*.

Answer: $V = \pi r^2 h$

> **TI-Nspire Navigator Opportunity:** *Live Presenter and Quick Poll*
> **See Note 1 at the end of this lesson.**

Move to page 2.2.

2. On Page 2.2, a cylinder with a cone on top is shown. Notice that the cylinder and the cone have the same radius and height. Grab the open circle, and move it in the direction of the arrow until all of the water has been drained from the cone into the cylinder.
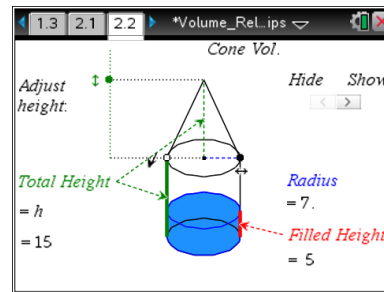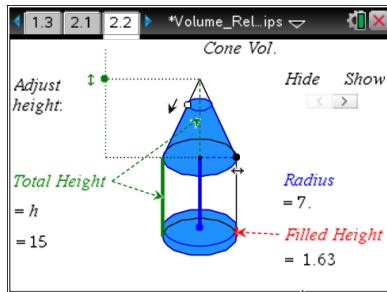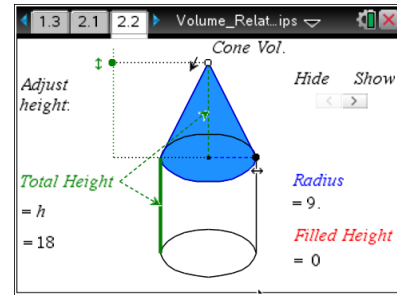
a. The first row of the table below has been completed for you. Complete the rest of the table by:
    • changing the radius and height,
    • recording the filled height of the cylinder, and
    • completing the calculation.
    To change the height, grab the solid dot next to the words *adjust height*. To change the radius, grab the solid dot on the end of the radius.

    <u>Answer:</u>

Total Height of the Cylinder and Cone	Radius of the Cylinder and Cone	Filled Height of the Cylinder	Filled Height ÷ Total Height (as a simplified fraction)
18	9	6	$\frac{6}{18} = \frac{1}{3}$
15	9	5	$\frac{5}{15} = \frac{1}{3}$
15	10	5	$\frac{5}{15} = \frac{1}{3}$
21	12	7	$\frac{7}{21} = \frac{1}{3}$

b. Look for a pattern in the table in part a. If a cone has radius 12 and a height of 18, predict the filled height of the cylinder and the quotient of the filled height and the total height of the cylinder.

    <u>Answer:</u> 6; $\frac{6}{18} = \frac{1}{3}$

c. Check your prediction in part b by changing the dimensions of the cylinder. How did your prediction compare with the actual volume of the cylinder? If you prediction is different than the actual value, explain why.

Answer: Depending on their answers to part 2b, students will respond that it is the same or different based on what they see on the calculator.

d. Look back at the table, and review your findings. Predict the filled height and the quotient of the filled height and the total height of a cylinder with radius 11 and height 21. Write your predictions below.

Answer: 7; $\dfrac{7}{21} = \dfrac{1}{3}$

e. Using the pattern you found for the filled height divided by the total height, what do you think is the relationship between the volume of the cone and the volume of the cylinder?

Answer: The volume of the cone is one-third the volume of the cylinder.

f. What do you think the volume of a cone is with radius 11 and height 18?

Answer: 726π

g. Click Show **cone vol** to see if your prediction in f is correct. Were you correct? If not, explain why.

Answer: Depending on their answers to part 2f, students will respond yes or no based on what they see on the calculator.

h. Use your findings about the relationship between the volume of a cone and the volume of a cylinder to write a formula for the volume of a cone with radius *r* and height *h*.

Answer: $V = \dfrac{1}{3}\pi r^2 h$

TI-Nspire Navigator Opportunity: *Live Presenter and Quick Poll*
See Note 2 at the end of this lesson.

Move to page 3.2.

3. On Page 3.2, there is a cylinder with a sphere above it. Notice that the cylinder and the sphere have the same radius and height. Grab the open circle, and move it in the direction of the arrow until all of the water has been drained from the sphere into the cylinder.
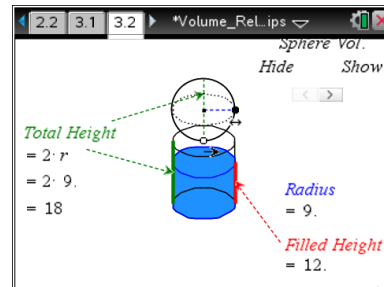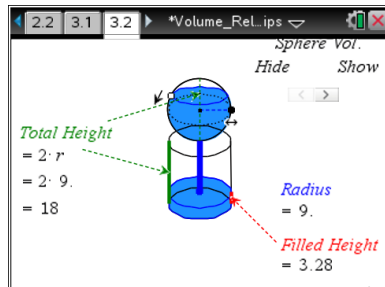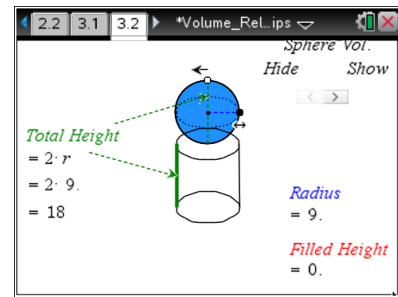
a. The first row of the table below has been completed for you. Complete the rest of the table by:
- changing the height,
- recording the corresponding volume, and
- complete the calculation.

To change the height, grab the radius of the sphere, and drag it left or right.

Total Height of the Cylinder and Sphere	Radius of the Cylinder and Sphere	Filled Height of the Cylinder	Filled Height ÷ Total Height (as a simplified fraction)
18	9	12	$\frac{12}{18} = \frac{2}{3}$
24	12	16	$\frac{16}{24} = \frac{2}{3}$
30	15	20	$\frac{20}{30} = \frac{2}{3}$
12	6	8	$\frac{8}{12} = \frac{2}{3}$

b. Using the pattern you found for the filled height divided by the total height (note: this is the same as the height or diameter of the sphere), what do you think is the relationship between the filled height of the cylinder and the height of the sphere?

Answer: The height of the sphere is two-thirds the filled height of the cylinder.

c. Using your answer from part b and the formula for the volume of a cylinder from Question 1e, write a formula for the volume of the sphere in relationship to the formula for the volume of a cylinder.

Answer: $V = \frac{2}{3}\pi r^2 h$

d. The height of the sphere is actually defined to be the diameter of sphere. Using this definition and the relationship between the diameter and the radius, rewrite and simplify the formula for the volume of a sphere in terms of only the value of the radius.

Answer: $V = \frac{2}{3}\pi r^2 h = \frac{2}{3}\pi r^2 d = \frac{2}{3}\pi r^2 \cdot 2r = \frac{4}{3}\pi r^3$

e. Using your formula from d above, determine the volume of a sphere of diameter 36.

Answer: 7776π

f. Check your answer in part e above, by changing the diameter of the sphere to 36 and clicking on the Show **sphere vol** arrow. Was your answer correct? If your calculation for the volume of the sphere is different, explain why.

Answer: Depending on their answers to part 3e, students will respond that it was the same or different based on what they see on the calculator.

TI-Nspire Navigator Opportunity: *Live Presenter and Quick Poll*
See Note 3 at the end of this lesson.

Extensions

1. Have students write what they found out about the relationships between the volumes of a cylinder, cone, and sphere in paragraph form.

2. Have students complete the following:
    a) Choose at least one of the volume formulas that you developed. Write a story problem that uses that formula in the solution.
    b) Write the steps and the calculations that would be used to solve your story problem.
    c) Students could trade their problems with other students.

3. Using the formula you have developed for finding the volume of a cylinder, what is the relationship between the volume of a given cylinder to the volume of a cylinder whose radius is double (or triple, or quadruple) that of the given cylinder? What if the height is doubled (or tripled or quadrupled)?

4. Using the formula you have developed for finding the volume of a cone, what is the relationship between the volume of a given cone to the volume of a cone whose radius is double (or triple, or quadruple) that of the given cone? What if the height is doubled (or tripled or quadrupled)?

5. Using the formula you have developed for finding the volume of a sphere, what is the relationship between the volume of a given sphere to the volume of a sphere whose radius is double (or triple, or quadruple) that of the given sphere?

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

*   The relationship between the formulas the volume of a cylinder, cone, and sphere.
*   How to use the formulas the volume of a cylinder, cone, and sphere.

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Note 1

Question 1, Live Presenter and Quick Poll

Allow a student (or students) to demonstrate the answers they obtained to one row of the table. Send an Open Response (or Multiple Guess) Quick Poll, asking students to submit their answers to 1b and 1d. Send an Open Response Quick Poll, asking students to submit their answer to 1e.

Note 2

Question 2, Live Presenter and Quick Poll

Allow a student (or students) to demonstrate the answers they obtained to one row of the table. Send an Open Response Quick Poll, asking students to submit their answers to 2b, 2d, 2e, and 2h. Send an Open Response (or Multiple Guess) Quick Poll, asking students to submit their answer to 2f.

Note 3

Question 3, Live Presenter and Quick Poll

Allow a student (or students) to demonstrate the answers they obtained to one row of the table. Send an Open Response Quick Poll, asking students to submit their answers to 3b, 3c, and 3d. Send an Open Response (or Multiple Guess) Quick Poll, asking students to submit their answer to 3e.