Conics in Winter

Student Activity

Name ___________________

Class ___________________

Conic Equations in Polar Notation

A conic is defined as the locus of points in a plane whose distance from a fixed point (focus) and a fixed line (directrix) is a constant ratio. This ratio is called the *eccentricity*, *e*, of the conic. The polar notation for the ellipse, hyperbola, and parabola is given by the equation:

$$r = \frac{e \cdot d}{1 \pm e \cdot \cos(\theta)}, \text{ or } r = \frac{e \cdot d}{1 \pm e \cdot \sin(\theta)}$$

where e is the eccentricity and d is the distance from the origin to the directrix.

Which Conic is It?

It seems impossible that this one equation can be manipulated into three of the conic sections, but it is true. To observe this, store 2 as **D** and then store different numbers as variable **E** and observe what happens to the graph for each value of **E**. Use positive and negative numbers and numbers between 0 and 1.

What values of e result in a(n):

- Ellipse?

- Hyperbola?

- Parabola?

The *d* Variable

What about the distance of the point from the directrix, *d*? How does this control the graph of the equation? Store 1 as **E**. Then store different values for the variable **D**. What happens to the graph?

Then change the value of **E** to experiment with other conic sections and summarize your results below.

The Other Stuff

Experiment with the formula. What happens if you change the plus sign in the denominator to a minus sign?

What happens if you use the sine function instead of the cosine function?

Experiment with other conic sections and summarize your results below.

Extension – The *a* Variable

What happens if a phase shift of *a* is added to the equation? This situation can be represented by the following equation:

$$r = \frac{e \cdot d}{1 \pm e\cos(\theta - a)}$$

What does the variable *a* control? Store 1 as **E**, 2 as **D**, and choose different values to store for the variable **A***.* What happens to the graph?

Experiment with other conic sections and summarize your results below.

Exercises

Determine the conic section for each equation listed below

1. $r = \frac{10}{1+3\cos(\theta-5)}$ **2.** $r = \frac{3}{1-\sin(\theta-6)}$ **3.** $r = \frac{20}{1-0.5\cos(\theta-2)}$