Limits of Indeterminant Forms

TEACHER NOTES

About the Lesson

In this activity, students will graph $f(x) = \frac{\sin x}{x}$ in order to visually determine the limit as $x \to 0$. They will confirm the answer numerically by tracing left and right limit points on a graph and looking at values in a table. As a result, students will:

- Graphically determine limits of various functions.
- Numerically determine limits of various functions.

Vocabulary

- graphical limit
- left-hand limit
- right-hand limit

Teacher Preparation and Notes

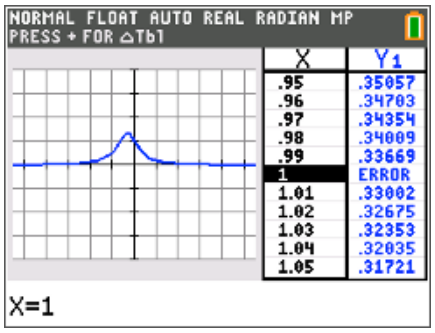
- With the accompanying worksheet, this TI-84 activity has step by step instructions so students can explore sin(*x*)/*x* graphically, numerically, and algebraically.
- A connection to the formal limit notation is considered to connect formal mathematical notation to visual representation.
- Before the activity begins, have students complete the following:
  - Press mode and set **Graph** to **FUNCTION** and **Angle** to **RADIAN**.
  - Turn the functions in y= off or clear them.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

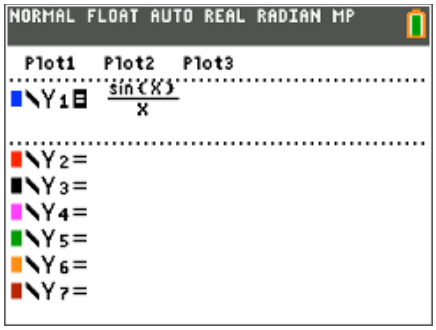
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

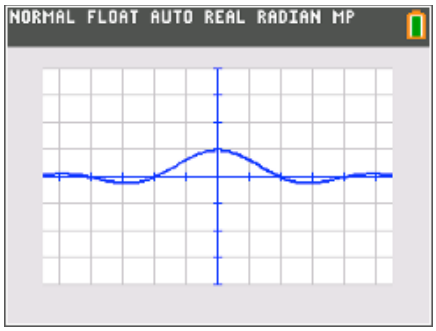
- Limits_of_Indeterminate_Forms_Student.pdf
- Limits_of_Indeterminate_Forms_Student.doc

Problem 1 – Graphical Limit

At the beginning of the activity, the student is introduced/reminded of the concept of indeterminate. Have them consider the value of sin(0) in order to understand that $Y_1(0) = \frac{\sin(0)}{0} = \frac{0}{0}$.

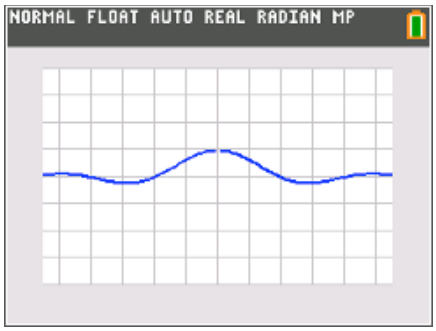


Students are to graph the function by typing $\frac{\sin(x)}{x}$ next to **Y1** in the [y=] screen. Viewing the graph, they need to quickly approximate **Y1**(*x*) as *x* approaches 0. The tick marks occur every 1 unit on the *y*-axis.



By turning off the axes (and grid if necessary), students can see the discontinuity of the function.

Trace is then used to see the values that correspond to the graph. Values can be inputted while Trace is active by simply typing a number like 0.1.



1. According to the graph, approximately what value does **Y1**(*x*) appear to equal as *x* approaches 0?

Answer: 1

2. Press trace. Examine points in the neighborhood of $x = 0$.

a. Type 0.1 enter. Then type 0.01 enter. What does the y-value equal as you move the point from the right toward $x = 0$?

Answer: 1

b. Repeat for –0.1, –0.01, etc. What does the y-value equal as you move the point from the left toward $x = 0$?

Answer: 1

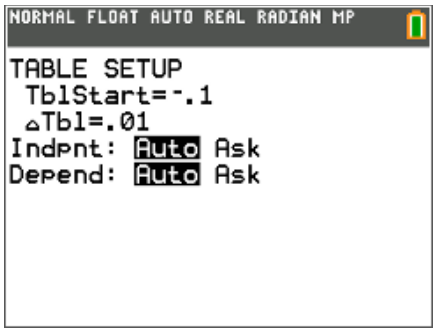
c. What happens when you type 0 enter? Why?

Answer: No y-value is returned because the function is not defined at $x = 0$. This is why limits are so valuable.

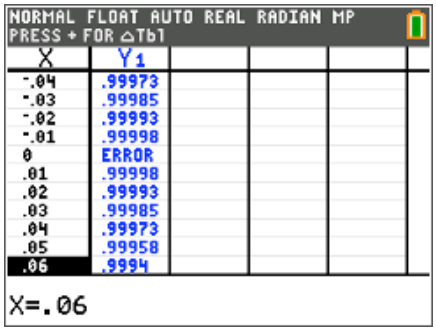
Teacher Tip: The last values for the x- and y-coordinates are automatically stored in case you want to recall the values of these coordinates for a calculation on the HOME screen. To see this press 2nd mode, then press X,T,θ,n and enter, then alpha 1.

Problem 2 – Numeric Limit

Students are now led to set up the table and investigate the values.



In order to emphasize that for a limit to exist it must be approaching the same value from both sides, students will move the curser to examine what happens on each side of zero.



3. Press 2nd graph to view the table of the function being graphed. Arrow down to observe what is happening to **Y1** as *x* approaches 0. To see more decimal places for **Y1**, arrow over to the **Y1** column and continue to arrow down and up.

a. Is **Y1** defined when $x = 0$? Explain.

Answer: No, because the calculator is unable to determine what $\frac{0}{0}$ is.

b. Does **Y1** appear to approach the same value from both sides of zero?

Answer: Yes, both sides appear to approach 1.

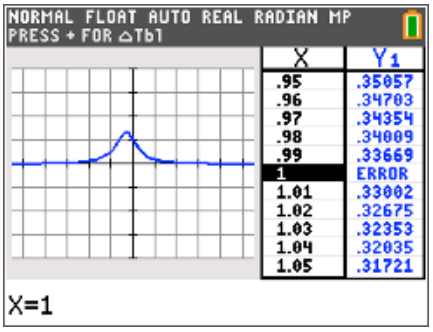
Problem 3 – Practice Problems

Students are given extra practice problems to complete using what they have learned in this activity. They can do the problems for homework or if there is extra time in class.

To use the split-screen, press mode and set **GRAPH-TABLE**. Press graph and 2nd graph to move between the graph and the table

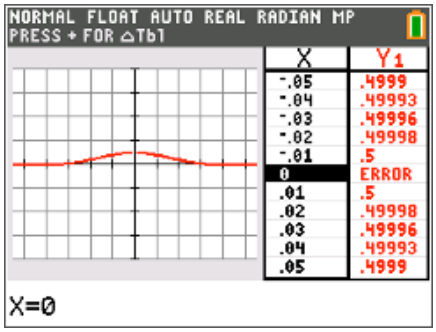
4. $\lim_{x \to 1} \frac{x-1}{x^3-1}$

Answer: $\frac{1}{3}$



5. $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$

Answer: $\frac{1}{2}$



6. $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$

Answer: ≈ 2.72