

Amortisation: What a killer

Student Worksheet

7 8 9 10 11 12



TI-Nspire CAS



Investigation



Student



90 min






Introduction

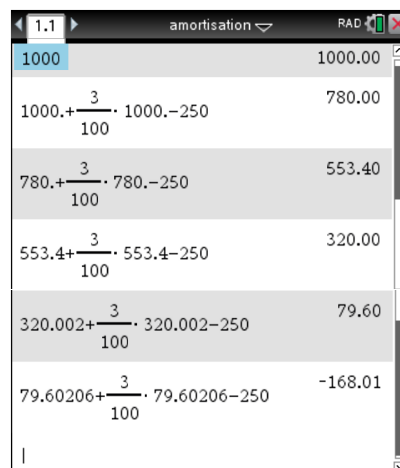
In its original meaning, *amortisation* means ‘to kill’, so the amortisation of a loan can be thought of as the process of killing off a debt. In this task we will look at how amortisation works for loans where the interest is calculated on the balance as it reduces over the life of the loan.

Recursion – baby steps!

Consider a small bank loan for \$1000. The interest rate for this loan is 12% p.a. compounding quarterly. At the end of each quarter, the interest is calculated and added to the value of the loan. In addition a repayment of \$250 is made. The calculator application of the TI-Nspire CAS can be used to find the reducing balance of this loan at the end of each quarter, as follows.

On the TI-Nspire CAS

- Press  > **New Document**, and then select **Add Calculator**
- Press  > **Settings** > **Document Settings** and change **Display Digits** to “Fix 2”.
- Press **OK** and then select **Current** to return to the **Calculator** page.
- Type **1000** and press 
- Type **ans + 3/100*ans – 250** and then press 
- Press  until the balance is a negative number.



| Calculation | Result |
|---|---------|
| 1000 | 1000.00 |
| $1000 + \frac{3}{100} \cdot 1000 - 250$ | 780.00 |
| $780 + \frac{3}{100} \cdot 780 - 250$ | 553.40 |
| $553.4 + \frac{3}{100} \cdot 553.4 - 250$ | 320.00 |
| $320.002 + \frac{3}{100} \cdot 320.002 - 250$ | 79.60 |
| $79.60206 + \frac{3}{100} \cdot 79.60206 - 250$ | -168.01 |

The results are shown in the screen shot at right. Note that in each new calculation of the quarterly balance, the calculator replaces ‘ans’ with the previous balance (i.e. the previous ‘answer’).

Question 1.

Answer the following questions about the above loan repayment scenario.

- How many quarterly repayments of \$250 are needed to amortise the loan?
- By how much could the final repayment be increased, so that the loan can be *amortised* after 4 payments?
- Explain why making four quarterly repayments of \$250 and a fifth payment at the end of fifth quarter \$79.60 would not amortise the loan.

- d) If V_n and V_{n+1} are used to represent successive loan balance amounts at the end of each quarter, write a recurrence relation that describes how successive balances are calculated.

Creating an amortisation table with “amortloan”

The previous amortisation example can be analysed further using a short program entitled “amortloan”.

On the TI-Nspire CAS

- Open the TI-Nspire file “amortloan.tns”
- Press and select **amortloan()** to run the program
- Enter the following values when requested.
 - Loan value (\$) = **1000**
 - Interest rate (%p.a.) = **12**
 - No. of payments to show = **4**
 - Payments (\$) = **250**
 - Cpy/Ppy = **4**
 - Adjust final payment (y/n)? **y**

Observe that the program has created a table, and provided two options for fully amortising the loan (if relevant to the scenario).

- An *adjusted* (increased) 4th payment, **or**
- An *extra* (5th) payment

Program output from “amortloan”

```

1.1 amortloan()
amortloan()
Loan value ($) = 1000
Interest rate (% p.a.) = 12
No. payments to show = 4
Payment ($) = 250
Cpy/Ppy = 4
["num" "pmt" "int" "p_red" "bal"]
0. 0. 0. 0. 1000.
1. 250. 30.4 220. 780.
2. 250. 23.4 226.6 553.4
3. 250. 16.6 233.4 320.
4. 250. 9.6 240.4 79.6
Adjust final payment?(y/n) y
Adjusted Amort. Table (last line only)
["num" "pmt" "int" "p_red" "bal"]
4. 329.6 9.6 320. 0.
...or if extra payment (No. 5.)
= $79.6 + (12. % / 4. ) / 100 * $79.6 = $81.99
  
```

What is “principal reduction”?

There is a column in the table that needs further explanation. The “**p_red**” column refers to “*principal reduction*”, which is the amount by which the original loan value (or principal) has been reduced in each quarter. As you can see from the table, the principal reduction is calculated as the difference between the repayment and the value of the interest in that quarter. In general, for each time period:

$$\text{principal reduction} = \text{repayment} - \text{interest}$$

Question 2.

How does the value of ‘principal reduction’ change as the loan repayment progresses over time? Explain why this change occurs.

Further analysis of the amortisation table

Once the program has been run, each column of values in the amortisation table is stored for further analysis. Press **VAR** to access them at any time. Here's what each variable represents.

- **num** number of loan balances over loan period
- **pmt** fixed repayment
- **interest** interest payment over the loan period
- **prinreduct** principal reduction amount over loan period
- **balance** loan balance over the loan period
- **amorttab** Final amortisation table
(including adjusted final payment if relevant)

| | | | | | |
|-------------------|---------------------------------|--------|-------|---------|---------|
| <i>num</i> | { 1.00,2.00,3.00,4.00 } | | | | |
| <i>pmt</i> | { 250.00,250.00,250.00,329.60 } | | | | |
| <i>interest</i> | { 30.00,23.40,16.60,9.60 } | | | | |
| <i>prinreduct</i> | { 220.00,226.60,233.40,320.00 } | | | | |
| <i>balance</i> | { 780.00,553.40,320.00,0.00 } | | | | |
| <i>amorttab</i> | "num" | "pmt" | "int" | "p_red" | "bal" |
| | 0.00 | 0.00 | 0.00 | 0.00 | 1000.00 |
| | 1.00 | 250.00 | 30.00 | 220.00 | 780.00 |
| | 2.00 | 250.00 | 23.40 | 226.60 | 553.40 |
| | 3.00 | 250.00 | 16.60 | 233.40 | 320.00 |
| | 4.00 | 329.60 | 9.60 | 320.00 | 0.00 |

The screen shows the output when **var** is pressed, and each of the variables is selected.

[Note: If you select "**amortloan**" it will just relaunch the program]

If you have not yet done so, type "**amortloan()**" to run the program. Enter the values as per the example on page 2, and type 'y' to ask the program to adjust the final payment for full amortisation in 4 payments. Then use the **var** key to verify that you get the same values as shown on the screen above right.

Now answer the following questions.

Question 3.

Type **sum(interest)** and then **enter**. What was the result, and what does it represent?

Question 4.

Type **sum(prinreduct)** and then **enter**. What was the result, and what does it represent?

Question 5.

Type **sum(pmt)** and then **enter**. What was the result, and what does it represent?

Question 6.

Use the program to find out how the number of payments required to amortise the above loan if the payment was only \$80/quarter.

Keeping it real – amortising a housing loan

Of course most loans are for greater amounts of money and take a lot longer to pay off than our simple example. Consider a typical home loan, which might be for \$300 000 with an interest rate 4.75% p.a. (compounded monthly), and is to be amortised over 20 years.

Question 7.

Run the program and try a monthly repayment of \$1000. Explain what happens to the loan balance in this case. Create a scatter plot of the loan balance (**balance**) vs. the number of repayments (**num**) to assist your analysis.

Question 8.

Run the program and try 3 other fixed monthly repayments for the loan above. Complete the following table to assess each repayment plan.

| <i>Option</i> | <i>Monthly repayment</i> | <i>How long for amortisation?</i> | <i>Total Interest paid?</i> | <i>Comment</i> |
|---------------|--------------------------|-----------------------------------|-----------------------------|----------------|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |

Question 9.

Use the program to find the least monthly repayment (correct to nearest dollar) that will fully amortise the above loan in exactly 20 years ($12 \times 20 = 240$ payments).

Question 10.

Use the program to find the monthly repayment that will mean that you will always have a loan balance of \$300 000 (hint: this is called an 'interest only' loan).

Checking with the Finance Solver

[This section makes the assumption that you are already familiar with the Finance Solver, and the meaning and use of the various parameters].

The purpose of the program was to 'look inside' the amortisation process. Now that you have used the TI-Nspire CAS to experiment with the effect of various amortisation parameters, let's check with the inbuilt finance tools available on your calculator. In many instances, such solvers are more efficient in finding the 'best' answer, or determining the consequences of changing a parameter on the amortisation time or related costs. The Finance solver can be accessed by pressing **menu** > **Finance** > **Finance Solver**.

Question 11.

Use the Finance solver to check your answer to Question 9.

Question 12.

In the same loan scenario as above, how much time would be saved by paying half of \$1939 (\$969.50) every fortnight, instead of \$1939 every month (the compounding period is still monthly)

Footnote: the 'amortTbl' command

Finally, there is a built-in function for generating amortisation tables as a matrix. It returns a matrix with columns in this order: Payment number, amount paid to interest, amount paid to principal, and balance. Its syntax is as follows:

amortTbl(NPmt,N,I,PV,Pmt,FV,PpY,CpY,PmtAt,roundValue)

For example, the command to generate the amortisation table for Question 11 is as follows (see calculator manual for more detail!)

amortTbl(240,240,4.75,300000,-1939,0,12,12,0,2)

The screenshot shows a TI-Nspire calculator window titled 'amortloan'. The command entered is `amortTbl(240,240,4.75,300000,-1939,0,12,12,0,2)`. The output is a matrix with 9 rows and 4 columns:

| Payment number | Interest | Principal | Balance |
|----------------|----------|-----------|-----------|
| 0 | 0.00 | 0.00 | 300000.00 |
| 1 | -1187.00 | -752.00 | 299248.00 |
| 2 | -1185.00 | -754.00 | 298494.00 |
| 3 | -1182.00 | -757.00 | 297737.00 |
| 4 | -1179.00 | -760.00 | 296977.00 |
| 5 | -1176.00 | -763.00 | 296214.00 |
| 6 | -1173.00 | -766.00 | 295448.00 |
| 7 | -1169.00 | -770.00 | 294678.00 |
| 8 | -1166.00 | -773.00 | 293905.00 |