

Activity 17

Fundamental Theorem of Calculus

Introduction

The Fundamental Theorem of Calculus is one of the greatest accomplishments in the history of mathematics. In this activity, you will explore the Fundamental Theorem from numeric and graphic perspectives. The version of the Fundamental Theorem covered here states that if f is a function continuous on the closed interval $[a, b]$, and x is any number in $[a, b]$, then the function

$$F(x) = \int_a^x f(t) dt$$

is an antiderivative for f . Another way of expressing this result is

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

Please refer to the table in Appendix B for the program commands.

Exploration

Define **Y1** and **Y2** in the **Y=** editor, and set up your viewing window as shown.

<pre> Plot1 Plot2 Plot3 \Y1=2cos(X^2/3)- .5 \Y2=fnInt(Y1(T), T, 0, X, .1) \Y3= \Y4= \Y5= </pre>	<pre> WINDOW Xmin=0 Xmax=5 Xscl=1 Ymin=-3 Ymax=3 Yscl=1 Xres=1 </pre>
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Objectives

- Explore the connections between an accumulation function, one defined by a definite integral, and the integrand
- Discover that the derivative of the accumulator is the integrand

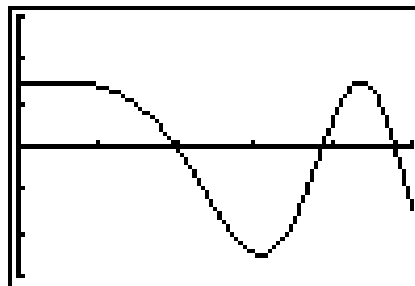
Materials

- TI-84 Plus / TI-83 Plus

Set your graphing handheld to radian mode. Note that **Y2** is a function defined by a definite integral. That is,

$$Y_2(X) = \int_0^X 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt$$

Also note that **Y2(X)** measures the accumulated *signed* area between the graph of **Y1(X)** and the x -axis from 0 to x . Make sure that **Y1** is the only function selected for graphing, and then view the graph. Your graph should look like the one shown.



If your graph is different, check that you have entered **Y1** correctly, that your window is correct, and that you are in radian mode.

Now create two lists. The first list, **L1**, stores inputs for the function **Y2(X)**. Those inputs will consist of x -values used in the graph of **Y1**. Use the graphing handheld's **seq** command to generate the list.

- Select **5:seq(** from the **LIST OPS Menu**.
- Fill in the arguments as shown on the screenshot at right.
- To get the variable ΔX , select **8:DX** from the **VARS (1:Window) X/Y** secondary menu.
- The **STO►** command is on the keypad above the **ON** key.
- The list variable, **L1**, is entered with **2nd [1]**.

```
seq(T,T,0,5,2*ΔX
)→L1
(0 .1063829787 ...
```

The first argument of the **seq** command, T in this case, is the expression used to generate the sequence. The second argument, again T , is the variable in that expression. The third, fourth, and fifth arguments represent the initial value, final value, and step size for that variable, respectively.

After producing the list of inputs, **L1**, evaluate **Y2** at each of those inputs. Store the list of outputs in **L2**. Do this by applying the function **Y2** to the list of inputs, **L1**. See the screen at right.

It takes time for all the integrals to be evaluated.

```
seq(T,T,0,5,2*ΔX
)→L1
(0 .1063829787 ...
Y2(L1)→L2
(0 .1595741653 ...
```

Each element of the output list, **L2**, results from the evaluation of a definite integral. For example, when the second element of **L1** is plugged into **Y2**, the value of

$$\int_0^{0.10638} 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt = 0.15957$$

is obtained, which is the second element of **L2**.

1. Explain why the first value of **L2**, which results from evaluating **Y2(0)**, is 0.

Graph the integrand function, **Y1(X)**. Use the graph to answer Questions **2** and **3**.

2. Explain why

$$Y2(0.10639) = \int_0^{0.10638} 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt$$

is greater than 0.

3. Why is the third value in **L2** greater than the second value?

$$\int_0^{0.21277} 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt > \int_0^{0.10638} 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt$$

Select **1:Edit** on the **STAT Menu** to view the table of inputs and outputs, **L1** and **L2**, respectively.

4. Look again at the values for **L2**, the outputs of the area accumulation function. For what value of x does

$$Y2(X) = \int_0^x 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt$$

reach its first local maximum? Approximate your answer.

5. Go back to the graph screen, and select **2:zero** from the **CALC Menu**. Find the smallest positive zero for **Y1(X)**. Note that **Y1(X)** changes sign from positive to negative at this point.

6. Explain why the twenty-first value of **L2** is smaller than the twentieth value.

$$\int_0^{2.1277} 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt < \int_0^{2.0213} 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt$$

(Keep in mind that **Y2(X)** is measuring *signed* area.)

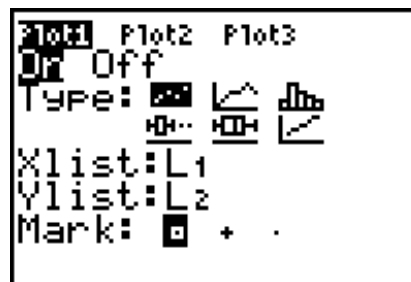
7. Explain why your answers to Questions 4 and 5 are approximately the same.

8. For what value of x does

$$Y2(X) = \int_0^X 2 \cos\left(\frac{x^2}{3}\right) - 0.5 dt$$

reach its first local minimum?

Define **PLOT1** as a scatter plot of **L2** versus **L1** as shown in this screenshot. Select **PLOT1** and **Y1** for graphing.



9. Select **2:zero** from the **CALC Menu**, and find the value of X at which **Y1(X)** changes sign from negative to positive.

10. Compare the answers to Questions 8 and 9. Explain why your answers are similar.

11. Find the x -coordinate of the largest positive root (x -intercept) of $Y_1(X)$ in the initial viewing window established in this activity.

12. What happens with $Y_2(X)$ at the point found in Question 11?

(Verify your answer numerically by looking at the table of values for $Y_2(X)$ in L2.)

Go back to the $Y=$ editor, and define $Y_3 = \text{fnInt}(Y_1(T), T, 1, X)$, thus changing the lower limit of integration from 0 to 1. Then, evaluate Y_3 at all the inputs in L1, and store the results in L3. Again, it takes time for the integrals to be evaluated.

```
Plot1 Plot2 Plot3
\Y1=2cos(X^2/3)-.
5
\Y2=fnInt(Y1(T),
T,0,X,.1)
\Y3=fnInt(Y1(T),
T,1,X,.1
\Y4=
```

```
Y3(L1)→L3
(-1.477891797 -...
```

Define **PLOT2** as a scatter plot of **L3** versus **L1**, and select **PLOT1**, **PLOT2**, and **Y1** for graphing.

```
Plot1 Plot2 Plot3
Off Off
Type: [ ] [ ] [ ]
[ ] [ ] [ ]
Xlist:L1
Ylist:L3
Mark: [ ] [ ] [ ]
```

13. Why is the graph of **L3** versus **L1** below the graph of **L2** versus **L1**? (Remember, the only difference between the two is the lower limit of integration.)

14. Explain why **PLOT1** and **PLOT2** have the same shape.

Look at the table of values for **L2** and **L3**, and notice that the locations of the extrema are the same for both accumulation functions (as you would expect from an inspection of the scatter plots).

15. Write an equation with a derivative on one side that shows how **Y1(X)** and **Y2(X)** are related.