Transformations

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Teacher Notes & Answers

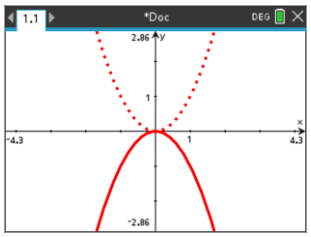
7 8 9 10 11 12

TI-Nspire™ | Investigation | Student | 50 min

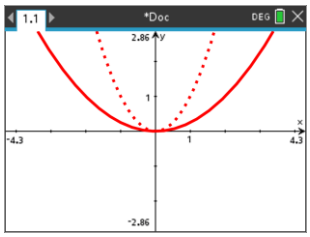
Introduction

Transformation means a change in form or appearance. Common transformations when dealing with functions include:

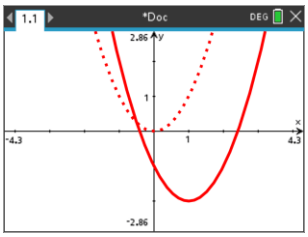
Reflection



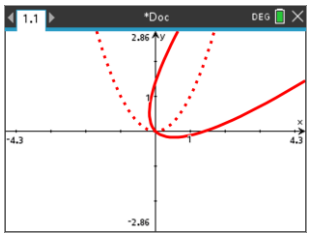
Dilation



Translation



Rotation



The aim of this activity is to provide an understanding of the algebra underpinning transformations. The technique involves the consideration of a single point and the effect it has on the general form or appearance of an entire family of points defined by a rule or function.
A video tutorial is available to help set up your TI-Nspire file.

https://bit.ly/TI-transformations

Set up

Open your "Transformations" document created using the video link above.

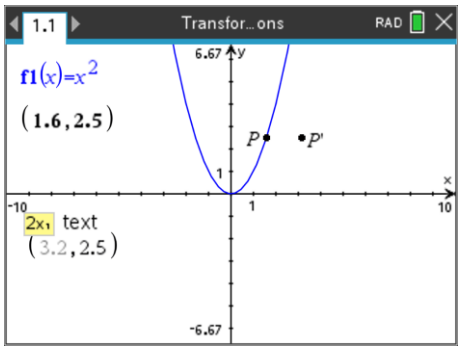
Point P (x_1, y_1) is on the parabola: $f_1(x) = x^2$

Point P has undergone a transformation such that:

$P'(x', y')$ such that: $x' = 2x_1$ and $y' = y_1$

The text tip on P′ provides the transformation details.

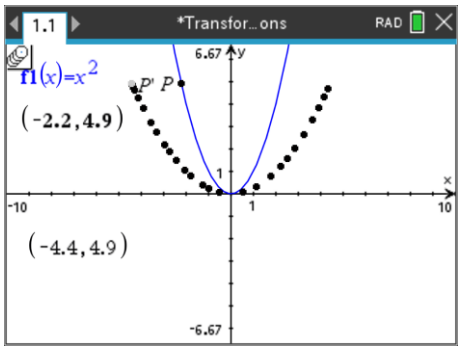
Edit the transformation for your point P′ to match these conditions.



Drag point P along the parabola and observe the coordinates of P′.

Point P′ is described as a dilation, "parallel to the x axis" or "away from the y axis" by a factor of 2.

In the screen opposite, the path of point P′ has been traced using the Trace (Geometry) tool.



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Determining Equations

Question 1.

a) Given $x' = 2x$, $y' = y$ and $y = x^2$, determine the relationship between x' and y'. Check your answer using your calculator and the corresponding transformation tools on the calculator.

Answer: $y' = \frac{(x')^2}{4}$ or $y = \frac{x^2}{4}$

b) Based on your answer to the previous question, describe the transformation from $y = x^2$ to $y = 4x^2$. Test your answer using your calculator and the transformations file.

Answer: Dilation parallel to the x axis (away from the y axis) by a factor of ½.

Question 2.

Edit the transformation for point P′ such that: $x' = x + 2$ and $y' = y$

a) Describe the location of point P′ in relation to P.
Answer: Point P′ is two units to the right (translation of 2 units in the positive x direction).

b) Determine the equation for the path of point P′.
Answer: $y = (x-2)^2$ or $y' = (x'-2)^2$

Question 3.

Edit the transformation for point P′ such that: $x' = x$ and $y' = y - 3$

a) Describe the location of point P′ in relation to P.
Answer: Point P′ is three units below point P (translation of 3 units in the negative y direction).

b) Determine the equation for the path of point P′.
Answer: $y' + 3 = (x')^2$ or $y = x^2 - 3$

Question 4.

Point P is dilated by a factor of 3 away from the x axis, then translated 2 units in the negative x direction. Use your calculator to observe the path of point P′ and determine the equation for $P'(x', y')$.

Answer: Transformations on x: $x' = 3x - 2$. Based on the order of operations, the dilation by a factor of 3 will occur first (as per description), followed by the translation of 2 units (in the negative x direction).

Equation: $y' = \frac{(x'+2)^2}{9}$ or $y = \frac{(x+2)^2}{9}$

Question 5.

Point P is translated by 2 units in the negative x direction, then dilated by a factor of 3 away from the x axis. Use your calculator to observe the path of point P′ and determine the equation for $P'(x', y')$.

Answer: Transformations on x: $x' = 3(x-2)$. Parenthesis must be used to order the transformations.

Equation: $y' = \left(\left(\frac{x'}{3}\right) + 2\right)^2$ or $y = \left(\frac{x}{3} + 2\right)^2$ Note that this can also be written as: $y = \frac{1}{9}(x+6)^2$

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Question 6.

Based on your answers to Questions 4 and 5, does the order of transformations matter?

Answer: Yes, the equations are very different. As the point is translated first the dilation 'from' the y axis is accentuated.

Question 7.

$P(x, y)$ is transformed such that $x' = x$ and $y' = 2y$, use your calculator to observe the path of point P′.

a) Determine the equation for $P'(x', y')$.

Answer: Equation: $\frac{y'}{2} = (x')^2$ or $y = 2x^2$

b) Write an equivalent transformation, based on your equation in part (a).

Answer: The equation shows that this is equivalent to a dilation away from the y axis by a factor of $\frac{1}{\sqrt{2}}$.

Question 8.

$P(x, y)$ is transformed such that $x' = x - 3$ and $y' = -y$, use your calculator to observe the path of point P′.

a) Determine the equation for $P'(x', y')$.

Answer: Equation: $-y' = (x' + 3)^2$ or $y = -(x + 3)^2$

b) State the corresponding transformations.

Answer: The graph $y = x^2$ has been reflected in the x axis and translated 3 units in the negative x direction.

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