

Tower of Hanoi

Student Activity

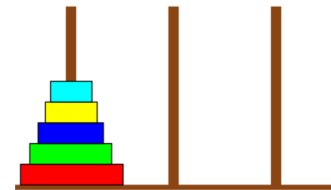
7 8 9 10 11 12



Introduction:

This puzzle was invented by a French Mathematician, Edouard Lucas in 1883. According to the legend, Brahmin priests moved one of the 64 disks every day according to the rules:

- Only one disk can be moved at a time;
- Disks must always be stacked on a pole;
- A large disk cannot be placed onto a smaller one.



The objective is to move all the disks from one pole to another. The legend also stated that the world would come to an end when the last disk was moved into place. If the legend is true, how long will it take to complete the puzzle? Surely if the legend was true, and the priests were still going, the world would have ended by now.

The aim of this investigation is to determine a rule that defines the number of moves to complete the puzzle consisting of a stack 'n' disks high.

Equipment:

For this activity you will need:

- TI-Nspire CX
- TI-Nspire CX file (tns): Tower of Hanoi

Instructions

Open the file:

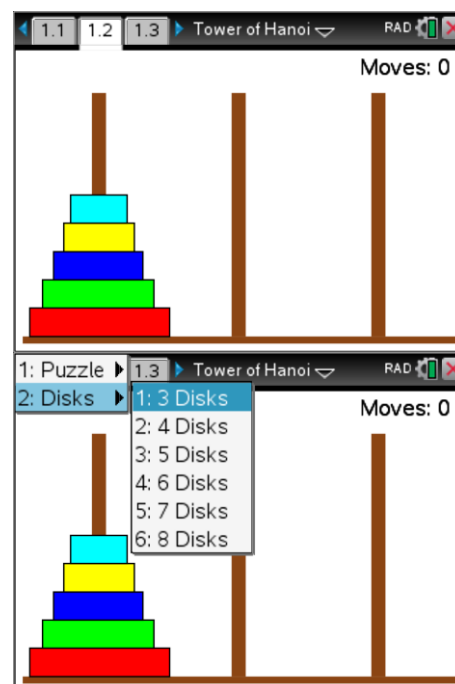
Tower of Hanoi

Navigate to the problem page (shown opposite).

Setting the number of disks

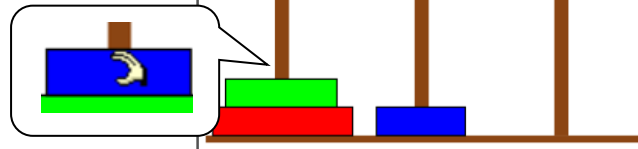
Use the [menu] key and set the number of disks to three.

- ✎ The single and two disk problems will be determined by reasoning rather than modelling.



Moving Disks

Move the mouse over the top disk, the mouse changes to an open hand. Press and hold the touchpad momentarily to grab the disk. Move the disk to a new column, press and hold the touchpad momentarily to release the disk.

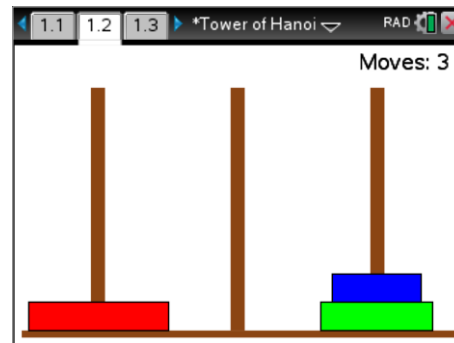


Moves

The number of moves is automatically recorded in the top right corner of the screen.

✎ Once a move has been made, you can't take it back.

Solve the three disk puzzle and make a note of the number of moves.



Entering Data

Navigate to the spreadsheet on page 1.3

Enter the numbers 1 to 8 in column A.

Leave the first two entries in column B blank. These will be included by reasoning and logic later. Record the minimum number of moves used to solve the three disk problem in cell B3.

	A	B	C	D
	disks	moves		
1	1			
2	2			
3	3			
4	4			
5	5			

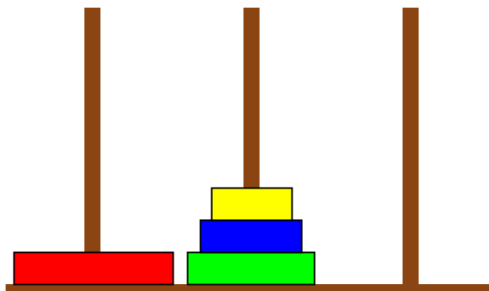
Questions

1. Use the puzzle to determine and record the minimum number of moves required to solve:
 - a. The three disk problem.
 - b. The four disk problem.

If the number of moves increases by the same amount each time a disk is added the puzzle, the relationship is said to be linear.

2. If the relationship between disks and moves is linear, how many moves should it take to solve the five disk problem?
3. Use the puzzle to determine the minimum number of moves required to solve the five disk problem and hence determine if the relationship is linear or non-linear.
Record your answer to the five disk problem in the spreadsheet.
4. Determine the number of moves required to solve the one and two disk problems. Explain how you determined these values. *Record the number of moves in the spreadsheet.*

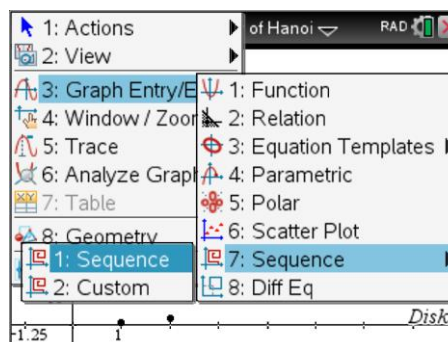
5. Navigate to the graph page, with your data for the one, two ... and five disk problems in the spreadsheet you will see five points on the graph. Do the points on the graph form a straight line or a curve?
6. The image shown opposite is a partial solution to the four disk problem. The large disk has not yet been moved.
 - a. How many moves have taken place so far?
 - b. Without referring to your previous data, how many moves are required to complete this puzzle?
 - c. Explain how you determined your answer to the previous question.



Developing a Rule

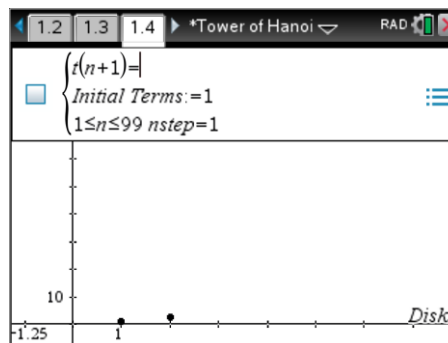
Navigate to the graph on page 1.4

Change the graph type to: **Sequence**



Change the sequence formula to your preferred notation. In the screen shown opposite $u(n)$ has been changed to $t(n+1)$.

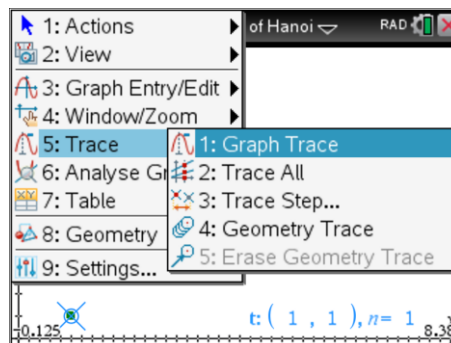
- Enter **your** formula using the appropriate notation. With the subject of the equation set to $t(n+1)$, the previous term can be referenced as $t(n)$.
- The next important component of the formula is to set the initial conditions. If the puzzle consisting of zero disks would require zero moves, therefore $t(0) = 0$. This would align to the *initial term* (moves) equal to zero and the *nstep* (n) also equal to zero. An alternative is to consider the puzzle to start with just one disk that requires just one move to solve. In this case: $nstep = 1$ and the initial term would be 1. Each of these entries can be edited by navigating to the corresponding value replacing it.



You can visually check your rule using the graph and numerically using the TRACE option. The graph being traced is indicated by the expression in the bottom right corner of the screen. "S1" indicates the scatter plot using the data from the spreadsheet, "t" indicates the sequence.

Another option to check the values for each point is to press Ctrl + T (Table) to toggle a table of values on/off.

Alternatively, navigate to page 1.5 (Calculator application) and type: $t(3)$ then press [Enter] to see the number of moves required to solve the three disk problem.



7. The legend states that the puzzle consisted of 64 disks. Use your formula to determine the number of days (and years) that would be required to solve the puzzle.
8. Another way to explore the rule for the total number of moves is to examine the number of moves for each disk in the puzzle. Complete the table below to help record the number of moves for each disk. Formulate a rule for the total number of moves using this method. (The one and two disk puzzles have already been completed)

Disk:	One	Two	Three	Four	Five	Six	Total
One disk puzzle	1						
Two disk puzzle	2	1					
Three disk puzzle							
Four disk puzzle							
Five disk puzzle							
Six disk puzzle							