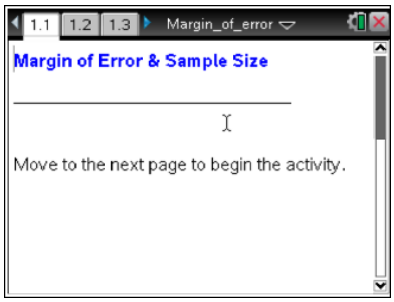
About the Mathematics

This activity investigates the margin of error for a confidence interval and the relationship between sample size and the margin of error.

This activity will give students the following opportunities:

1. Select an observed proportion of successes from a random sample chosen from an unknown population.
2. Examine simulated sampling distributions of proportions for samples drawn from known populations and decide whether the observed proportion seems likely to have come from this population; if so, a point is marked on a number line representing likely population proportions.
3. Repeat sampling for a sequence of known population proportions until an interval of successes—a confidence interval—is marked on a horizontal axis.
4. Express a confidence interval in terms of a sample proportion and the margin of error.
5. Experiment with changing the sample size for a given observed proportion and note the change in the confidence interval and consequently the margin of error.



TI-Nspire™ Technology Skills:

- Open a document
- Move between Pages
- Click on a minimized slider

Lesson Materials:
Margin_of_Error_&_Sample_Size.tns

Visit www.mathnspired.com for lesson updates.

The approach to confidence intervals in this activity is intended to develop conceptual understanding of a confidence interval for proportions. The activity provides a foundation for the formulas that are typically used to calculate confidence intervals.

Math Objectives

- Students will interpret a confidence interval for a population proportion as an estimated range of values that is likely to contain the actual population proportion.
- Students will estimate the margin of error for a generated confidence interval and express the confidence interval in terms of the margin of error.
- Students will recognize that as sample size increases, the margin of error decreases.
- Students will reason abstractly and concretely (CCSS Mathematical Practices).
- Students will use appropriate tools strategically (CCSS Mathematical Practices).

Prerequisite Knowledge

Students should be familiar with calculating a confidence interval (see the Statistics Nspired activity *Confidence Levels for Proportions*) and understand that the larger the confidence level, the larger the width of the interval. In addition, the Statistics Nspired activity *Standard Error and Sample Means* discusses the relationship between sample size and standard error (i.e., the larger the sample size, the smaller the standard error).

The Problem Situation

Students will investigate the questions:

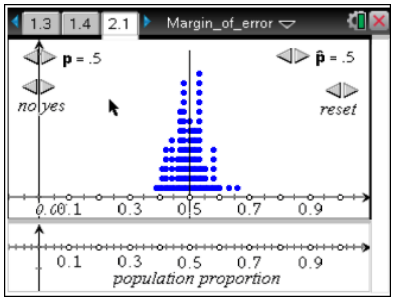
- How can a confidence interval be written using the language of margin of error?
- What is the relationship between the margin of error and the sample size?

The .tns document contains the following problem conditions:

- Confidence intervals are calculated as (statistic) ± (margin of error).
- The margin of error is dependent upon the confidence level and the sample size.

The task in this activity is to investigate which population parameters the sample proportion, *p*, is likely to have come from based on the sampling distributions of the population. The confidence interval students generate will be used to estimate the margin of error.

Using the Document



Page 2.1: Generating a Confidence Interval

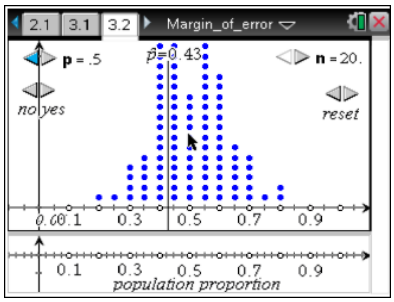
Clicking the top right arrow will select a value for $\hat{p}$ (p-hat), an observed proportion of successes from a sample from an unknown population. Clicking the top left arrow sets the proportion of successes for a known population, p.

A simulated sampling distribution of the proportion of successes for that population is displayed with a vertical line representing the observed proportion. When the sampling distribution seems likely to contain $\hat{p}$, the student clicks the yes arrow to place a point on the number line in the bottom work area.

Varying p to represent different known population proportions and repeating the 'yes' process will build a confidence interval on the number line that is likely to contain the true population proportion, p, for the observed sample proportion, $\hat{p}$. Students should consider how they might represent the confidence interval using inequalities and in particular, express the interval in terms of its center. A discussion with students about the calculation of the margin of error—the distance from the center of the confidence interval to each end of the interval—might be beneficial at this point in the lesson.

Students should be asked to express the confidence interval in the form, (statistic) ± (margin of error). In this investigation, the boundaries are estimated without an explicit definition of when an outcome is likely. Thus, using the observed proportion as the center of the interval might be slightly inaccurate. It is important to note that the margin of error will be an estimate because the confidence intervals are not calculated, but student-generated. Emphasize that a 5% margin of error does not mean the observed proportion is wrong by 5% but that the actual proportion is likely to be within 5% of the value of $\hat{p}$.

Page 3.2: Sample Size and Confidence Intervals



Although in practice, it is usually only feasible or possible to draw a single sample from any given population, the functionality of the arrow on Page 3.2 simulates the possibility of being able to change the sample size (20, 30, 40, …,100) for a fixed observed proportion, 0.43, from a random sample from an unknown population. This allows students to investigate the relationship between sample size and the margin of error.

Once again, students will generate a confidence interval. They should note the margin of error for each confidence interval generated for the sample size *n* and express the confidence interval in terms of the sample proportion and the margin of error.

The reset arrow will clear the existing interval and provide the opportunity to make and check conjectures about the margin of error as related to sample size.

Students should note that as the sample size increases, the margin of error decreases.

Note that this activity does not specify what is meant by "typical" but allows students to informally make a decision from the visual representation of the sampling distribution and where the observed proportion falls relative to the proportions of other samples from that population. Developing the concept of confidence *level* is explored in the Statistics Nspired activity *Confidence Levels for Proportions*.

Related Activities

- Statistics Nspired *Confidence Intervals for Means*
- Statistics Nspired *Confidence Intervals: 2-Sample Proportions*
- Statistics Nspired *Standard Error and Sample Meansh*