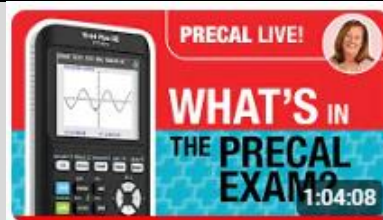




Thursday Night Precalculus Series


April 4, 2024

In this AP Precalculus Live session, we will explore questions which use technology to prepare for the AP Precalculus Exam.



Looking Ahead: Tips for the AP® Precalculus Exam

About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
 - <https://www.youtube.com/live/UCb-endHGIQ?si=uVBT4ePTjOH8Fvyz>
 - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the “Live Session” but can be enriched by that resource.*
- This session involves exploring a variety of questions that involve the use of the graphing calculator, such as:
 - Input output values and zeros of a function.
 - Regression.
 - Exponential models.
 - Logarithmic functions.
 - Piecewise defined functions.
- Students should be able to use the TI-Nspire to answer multiple choice and free response questions on the AP Precalculus Exam.
-  **Class Discussion:** Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:

- TI-Nspire document*
- Exam_Prep.tns
- Student document*
- Precal_problems_04_04
- Teacher document*
- problems_solutions_04_04
- YouTube
- <https://www.youtube.com/live/UCb-endHGIQ?si=uVBT4ePTjOH8Fvyz>
- **Documents and materials can be downloaded from this site.**

AP Precalculus Mathematical Practices

- 1.A: Solve equations and inequalities represented analytically, with and without technology.
- 1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.



- 2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
- 2.B: construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.

Source: AP Precalculus Course and Exam Description, The College Board

Problem 1.

The function f is defined by $f(x) = \frac{2x^3 - 3x^2 + 7}{x - 3}$. What input value(s) in the domain of f yields an output value of -5 ?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: On a calculator page, define the function f . Use the solve command (Menu > Algebra > Solve) or the zeros command (Menu > Algebra > Zeros). The graphing application could also be used. To display more decimal digits on the graphing screen, hover over the x-value and press the plus key.



Class Discussion:

Should we be concerned about what is happening with the graph to the right of 5?

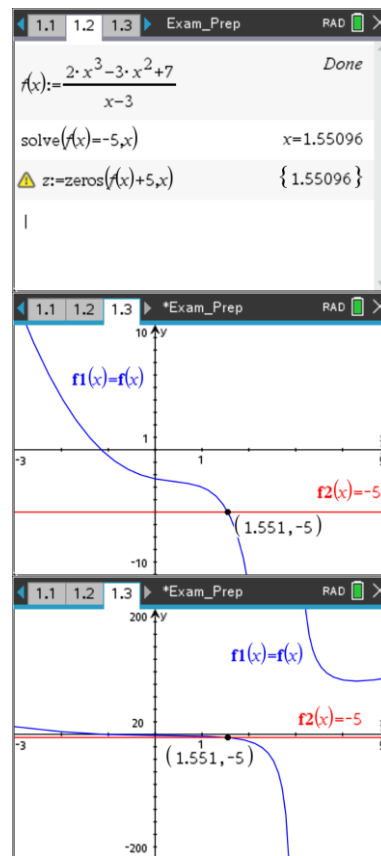
Possible Answers: Yes, if the student is using the graph of f to find the input when the output is -5 , then it would be a good idea to explore the graph by changing the window.



Class Discussion:

What is going on with this graph at $x = 3$?

Possible Answers: The graph has a vertical asymptote.





Problem 2.

The table shows values for a function f at selected values of x .

x	-2	-1	0	1	2
$f(x)$	-0.5	0.1	-2	0.5	10

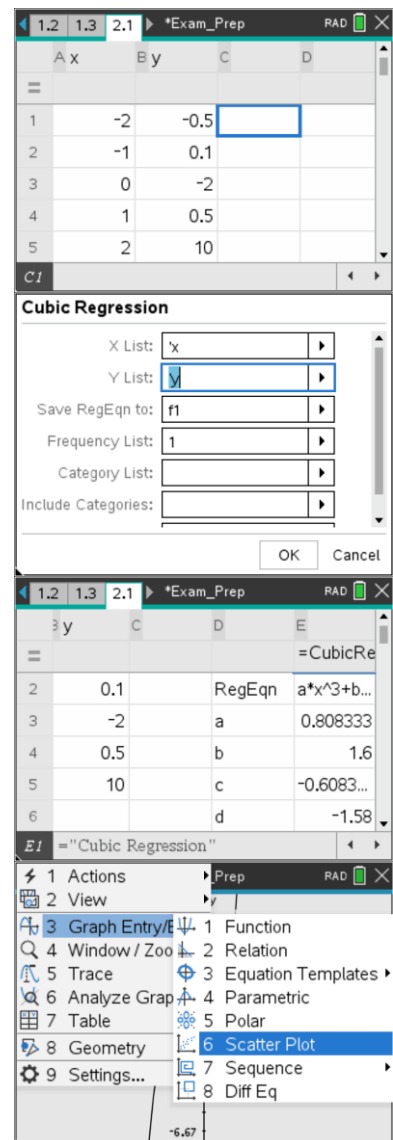
A cubic regression is used to model the function f . What is the value of $f(0.5)$ predicted by the cubic regression model?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: Add a Lists & Spreadsheet application. Name the lists x and y , then enter the data. The cubic regression may be run from the Lists and Spreadsheet application or a Calculator application. Go to an empty cell (C1) and select Menu > Statistics > Stat Calculations > Cubic Regression. Complete the prompts as shown to the right.

To graph both the regression and the scatter plot, add a graphing application. Graph $f1(x)$, then select Menu > Graph Entry/Edit > Scatter Plot.

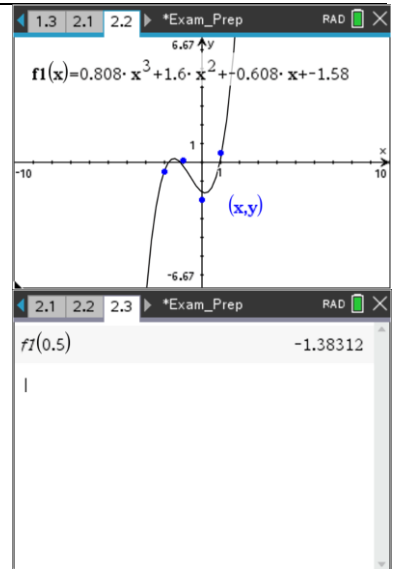




Class Discussion:

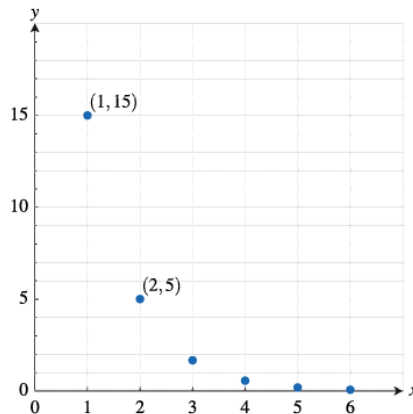
Does the model underestimate or overestimate the actual value at $x = -1$?

Possible Answers: The model predicts -0.18 , but the given value is 0.1 . The model predicts a value below the actual value, so it is an underestimate.



Problem 3.

A geometric sequence has the form $g_n = g_k \cdot r^{(n-k)}$. The graph of a geometric sequence, g_n , is shown in the figure.



What is the value of g_5 ?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 4.

The growth of bacteria in a culture is modeled by $y = 100e^{0.75t}$, where t is measured in days. At what time t is the number of bacteria approximately 1500?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

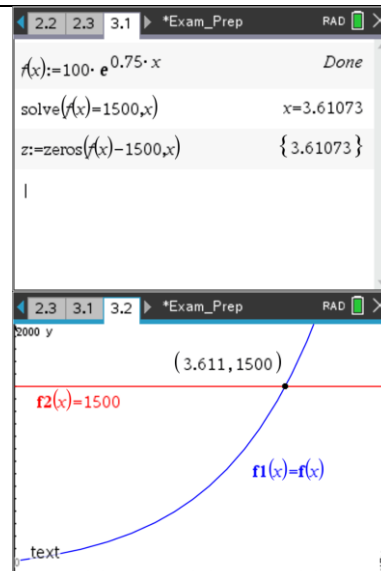


Technology Tip: On a calculator page, define the function f . Use the solve command (Menu > Algebra > Solve) or the zeros command (Menu > Algebra > Zeros). The graphing application could also be used. To display more decimal digits on the graphing screen, hover over the x-value and press the plus key.

**Class Discussion:**

How do students know that there is only one solution?

Possible Answers: This exponential function is increasing and concave up so there is only one intersection point.

**Problem 5.**

Consider the logarithmic functions f and g defined by $f(x) = \log_3(2.5x+1)$ and $g(x) = 3 - 2\log_3(1.4x-1)$. Find a zero of the function h defined by $h(x) = f(x) + g(x)$.

Sample Solution:

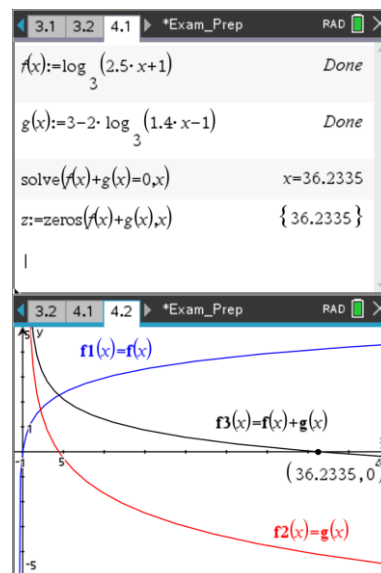
Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: On a calculator page, define the functions f and g . Use the solve command (Menu > Algebra > Solve) or the zeros command (Menu > Algebra > Zeros). The graphing application could also be used. To display more decimal digits on the graphing screen, hover over the x-value and press the plus key.

**Class Discussion:**

In the algebraic solution using the quadratic formula, there are two solutions. Why do we have only the one solution of 36.233?

Possible Answers: The quadratic formula solution of -0.366 is called an extraneous solution. The value of -0.366 is not in the domain of the function g .



**Problem 6.**

The function f is given by $f(x) = \cos(2.3x) - \sin(1.7x)$. The function g is given by $g(x) = e^{0.75x} - 2.5$. Find the input value such that $f(x) = g(x)$.

Sample Solution:

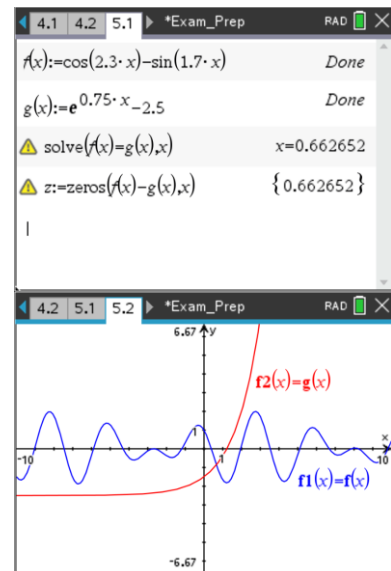
Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: On a calculator page, define the functions f and g . Use the solve command (Menu > Algebra > Solve) or the zeros command (Menu > Algebra > Zeros). The graphing application could also be used. Students may Zoom In to see the intersection point more clearly. To display more decimal digits on the graphing screen, hover over the x-value and press the plus key.

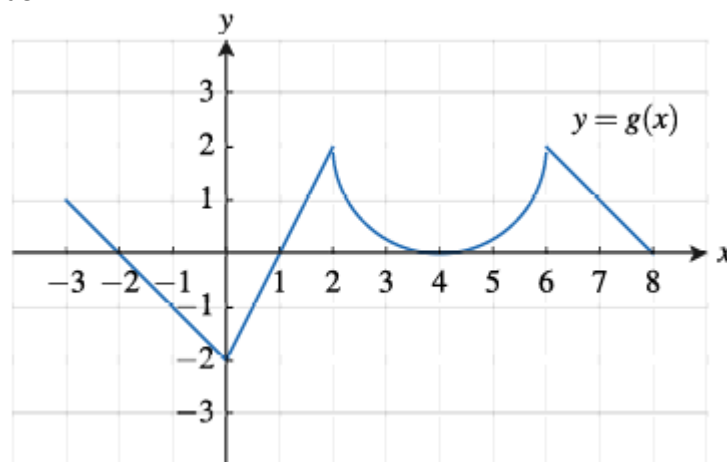
**Class Discussion:**

How do we use the features of an exponential function to be certain that there is only one intersection point?

Possible Answers: This exponential function g has a vertical translation down 2.5 units and is always increasing; therefore, g will only intersect the periodic function once.

**Problem 7. (A) – (C)**

The graph of the function g is shown in the figure and consists of three line segments and a semicircle with radius 2.





The function f is given by $f(x) = \frac{-3x^2 + 1.9x + 4.5}{x^3 + 2x^2 + 1}$.

- (A) (i) The function h is defined by $h(x) = (f \circ g)(x) = f(g(x))$. Find the value of $h(7)$, or indicate that it is not defined.
- (ii) Find all values of x for which $g(x) = -1$, or indicate there are no such values.
- (B) (i) Find all real zeros of f , or indicate there are no such values.
- (ii) Determine the end behavior of f as x increases without bound. Express your answer using the mathematical notation of a limit.
- (C) (i) Determine if an inverse function of g can be constructed for all values of x in the closed interval $[2, 6]$.
- (ii) Give a reason for your answer based on the definition of a function and the graph of g .

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Technology Tip: On a calculator page, define the function f .

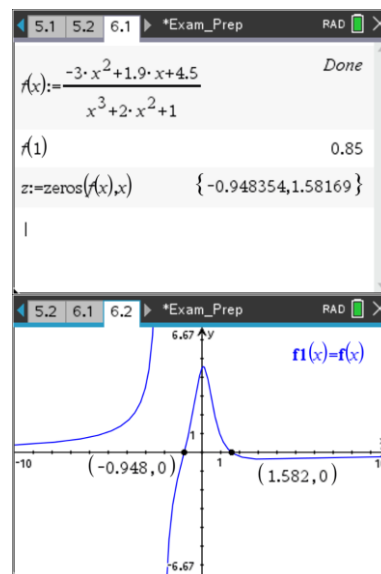
Find the value of the function on the calculator page. Use the graph to calculate the zeros of the function.



Class Discussion:

Why does this function's values approach zero as x increases without bound?

Possible Answers: If we look at the polynomials in the numerator and denominator, the cubic in the denominator dominates the quadratic in the numerator.





Note: The following problem 8 is not discussed in the video.

Problem 8. (A) – (C)

The cost of an Uber ride in Boston is modeled by the function C given by

$$C(m) = \begin{cases} a \cdot m + b \cdot m^2 & \text{if } 0 < m \leq 5 \\ d \cdot (m - 5) + 25 & \text{if } m > 5 \end{cases}$$

where m is measured in miles and C is measured in dollars. Two Uber riders reported that for $m = 1$ the cost was \$9.00 and for $m = 3$ the cost was \$21.00.

- (A) (i) Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $C(m)$.
- (ii) Find the values of constants a and b .
- (B) (i) Use the given data to find the average rate of change of the cost of a ride, in dollars per mile, from $m = 2$ to $m = 4$. Show the computations that lead to your answer.
- (ii) Interpret the meaning of your answer from (i) in the context of the problem.
- (iii) The two pieces of the function C connect at the transition point when $m = 5$. It is known that $\lim_{m \rightarrow 5} C(m) = 25$ and $C(6) = 27.5$. Consider the average rates of change of C from $m = 5$ to $m = p$ miles, where $p > 5$. Are these average rates of change less than or greater than the average rate of change from $m = 2$ to $m = 4$ miles found in (i)? Explain your reasoning.
- (C) (i) Using the model C to predict the cost of an Uber ride, what is the maximum amount a rider could pay? Explain your reasoning.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.



Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The tasks that students need to be able to perform using the TI-Nspire for the exam.

For more videos from the AP Precalculus Live series, visit our playlist

https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI

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