Introduction

Logarithmic equations are helpful in real applications whenever possible values cover extremely large ranges. A couple of examples to be considered are pH and sound intensity level.

- Sound intensities range from 10^{-12} watts per square meter W/(m^2) at the lowest limit for audible sound up to 10 W/m^2, where the a listener will begin to experience pain.
- In solutions of hydrochloric acid, hydrogen ion concentrations can range roughly from 10^{-20} moles per liter (M) up to nearly 13 M.

Problem 1 – Intensity of Sound

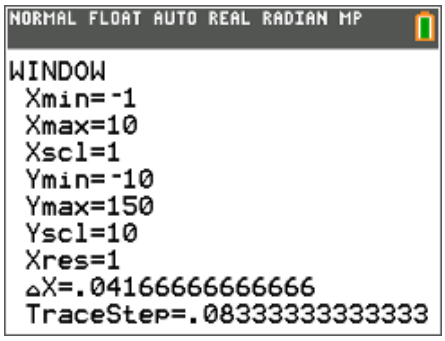
The following equation is used to convert the power level of sound to decibels (dB).

$$b = 10\log\left(\frac{I}{I_0}\right)$$

β is the intensity level of a sound wave, I is the intensity in $\frac{W}{m^2}$ and I_0 is a reference intensity, $\frac{10^{-12}W}{m^2}$, which corresponds roughly to the faintest sound that can be heard.

Using the information listed above, graph the equation $b = 10\log\left(\frac{I}{I_0}\right)$.

To better view the graph, press window and change the settings to those on the right.



1. Using the graph and the trace key, describe the features of this graph. (What happens to the graph as $x \to \infty$? What happens as $x \to 0$? Is the function increasing or decreasing? What happens when x is negative? Where does the function change rapidly? Where does the function change slowly?)

Below is a table of sound intensity levels for a variety of situations with distances away from the actual sound source.

Use the intensity equation and your calculator to complete the table.

Source	Power	Intensity
Jet engine (30 m away)	10^{2}	
Threshold of pain	10^{1}	
Pneumatic drill	10^{0}	
Rock concert (2 m away)	10^{-1}	
Niagara Falls	10^{-3}	
Hearing damage: long term	10^{-3}	
Busy traffic	10^{-5}	
Normal talking (1 m away)	10^{-6}	
Library	10^{-8}	
Leaves rustling	10^{-11}	
Auditory threshold	10^{-12}	

2. Which of the following event(s) will cause damage *only* if exposure is *long term*?

a. Normal talking

b. Niagara Falls

c. Busy traffic

d. Jet engine

3. Which of the following event(s) listed will result in hearing loss following short-term exposure?

a. Jet engine

b. Pneumatic drill

c. Busy traffic

d. Normal talking

4. Elevated trains, such as the "L" in Chicago, produce a great deal of noise. If the sound level recorded from one of these trains is 90 dB, use the intensity equation $b = 10\log\left(\frac{I}{I_0}\right)$ to find the power (I) in $\frac{W}{m^2}$.

Problem 2 – pH

PH is defined as the negative logarithm of the hydrogen ion concentration in molarity (H^+).

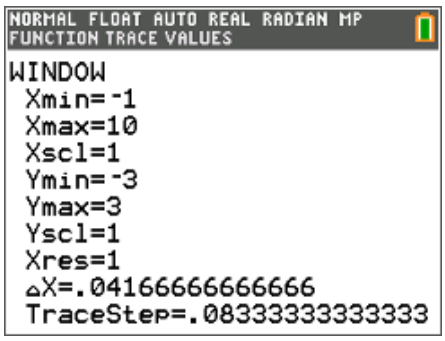
- A low pH implies a high degree of acidity and vice versa.
- The typical pH range is 0–14.
- pH = –log[H^+]

Graph this function on the following page. Let x = [H^+].

Sketch a graph of the function pH = –log[H^+].

Hint: Let x = [H⁺].

To better view the graph, press window and change the settings to those on the right.



5. Describe the features of this graph. (What happens to the graph as $x \to \infty$? What happens as $x \to 0$? Is the function increasing or decreasing? What happens when x is negative? Where does the function change rapidly? Where does the function change slowly?)

6. Why do negative values of x not make sense in the context of the pH equation?

Below is a table of pH data for various household substances. Complete the table by using your calculator to compute the concentration, $[H^+]$.

Source	pH	$[H^+]$
Battery acid	0	
Gastric fluid	1.2	
Lemon juice	2.3	
Carbonated beverages	2.9	
Vinegar	3	
Tomato juice	4.1	
Coffee	5	
Rain water	5.8	
Milk	6.6	
Distilled water	7	
Sea water	8	
Milk of magnesia	10.7	
Household ammonia	11.5	
Household bleach	12.6	
Lye solution	14	

7. Which of the following substances is most acidic?

a. Vinegar

b. Tomato juice

c. Rain water

d. Carbonated beverages

8. Which of the following is least acidic (or most basic)?

a. Sea water

b. Gastric juices

c. Milk

d. Distilled water

9. Do any of the values listed in the table surprise you? Which ones? Why?