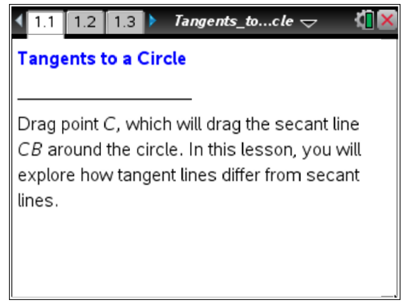
Tangents to a Circle



Math Objectives

- Students will define a tangent and recognize that a tangent is perpendicular to the radius of the circle at the point of tangency.
- Students will understand that two segments tangent to a circle from a common point outside the circle are congruent.
- Students will be able to prove that the tangent segments from an external common point are congruent.
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

- secant line
- tangent line
- point of tangency
- tangent segments

About the Lesson

- This lesson involves students looking at tangents and their properties.
- As a result students will:
  - Manipulate a point on a line to visualize when it is a secant line and when it becomes a tangent line to the circle.
  - Using a constructed tangent line, describe the relationships of a tangent line to a radius at the point of tangency.
  - Using two tangent lines intersecting outside a circle, discover the relationships of tangent segments.
  - Step through and justify a proof for tangent segments.

TI-Nspire™ Navigator™

- Quick Poll
- Live Presenter

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Tangents_to_a_Circle_Student.pdf
- Tangents_to_a_Circle_Student.doc

TI-Nspire document

- Tangents_to_a_Circle.tns

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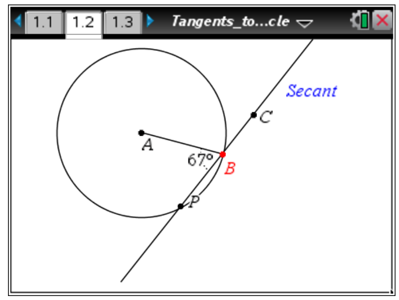
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (👆). Press ctrl 🖐 to grab the point and close the hand (✊) After the point has been moved, press esc to release the point.

Move to page 1.2.



1. a. As you drag point *C*, what happens to ∠*CBA*?

   Answer: The angle measurement increases and decreases.

   b. When points *P* and *B* are very close to each other, what is the measure of ∠*CBA*? What happened to point *P*?

   Answer: 90°. Point *P* is at the end of the radius $\overline{AB}$. Although *B* and *P* are distinct points, point *P* is being hidden by point *B*.

   c. When ∠*CBA* measures 0°, where is point *P* on the circle in relation to *B*?

   Answer: *P* is now on the opposite side of point *B*. The secant line goes through the center of the circle and $\overline{PB}$ is a diameter.

   d. When ∠*CBA* measures 90°, what has happened to the secant line?

   Answer: It is hitting the circle only in one point, thus becoming a tangent line. Students might also say something about the radius being perpendicular to the tangent line. Points *P* and *B* are coinciding but still distinct. A tangent does not pass through the interior of the circle.

Teacher Tip: You may want to review inscribed angles here as well. You also may want to discuss the definition of a tangent line.

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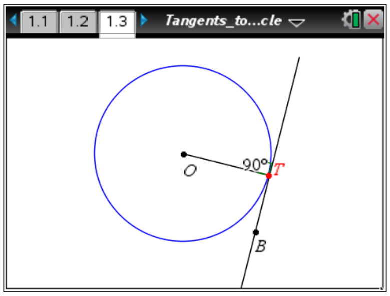
Teacher Tip: Students can also drag point *B*. If they do, and then create a tangent line as a preview to the next page, it might be difficult to drag point *B* to another place on the circle. They will have to tab one time to get to point *B*.

Move to page 1.3.



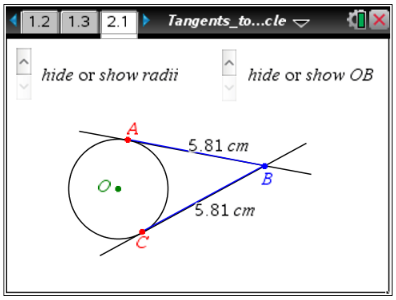
A tangent line has been constructed at point *T*. Drag point *B* to move the tangent line around the circle.

2. A tangent line intersects the circle in exactly one point, which is known as the point of tangency. How is a tangent related to the radius at the point of tangency?

   Answer: As the tangent is being dragged around the circle and the radius is rotating around the circle, the tangent line is still intersecting the circle at only one point. A tangent is perpendicular to the radius at the point of tangency.

Teacher Tip: This conjecture has important applications that are related to circular motion. How does a satellite stay in orbit? The satellite is pulled by gravity in a direction that is perpendicular to the direction of the satellite's velocity. The satellite's velocity is tangent to its circular orbit. The velocity vector is perpendicular to the force of gravity.

Move to page 2.1.



3. Drag point *B* and observe the tangent segments $\overline{AB}$ and $\overline{BC}$.

   a. What can you conjecture about the tangent segments $\overline{AB}$ and $\overline{BC}$?

      Answer: Tangent segments to a circle from a point outside the circle are congruent.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

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b. What happens to the tangent segments when *B* is inside the circle? Why?

Answer: The tangent segments disappear because the lines are no longer tangents.

> **Teacher Tip:** Discuss with students that tangent segments are congruent, not tangent lines.

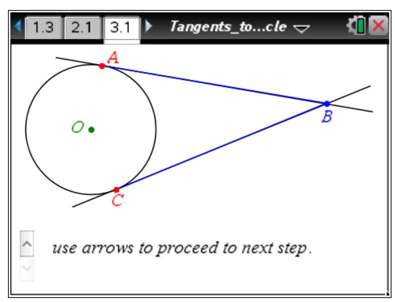
> **Teacher Tip:** What happens if *B* is on the circle? Although is it difficult to drag point *B* exactly on the circle, the **Redefine** tool could be used to move it to the circle. If this is done, all three points coincide.

c. Select to show the radii and $\overline{OB}$. Look at the triangles formed from the segments. What do you notice about ΔAOB and ΔCOB?

Answer: These triangles are both right triangles.

> **Teacher Tip:** This might be a good time to talk about the upcoming proof by looking at the two triangles. Students should notice that these triangles are both right triangles because of the tangent/radius relationship established earlier. They might say that they could use the Pythagorean Theorem to find the measure of the tangent segments. Then the triangles would be congruent by Side-Side-Side. Beware of rounding if students use the Pythagorean Theorem.

Move to page 3.1.



4. Prove that $\overline{AB} \cong \overline{CB}$.

   a. Select ∆ to draw $\overline{OA}$ and $\overline{OC}$. Press ∆ to show the next step. Why is $\overline{OA} \cong \overline{OC}$?

   Answer: $\overline{OA}$ and $\overline{OC}$ are radii of a circle and therefore congruent.

   b. Select to show the next step. Why is $\overline{OA} \perp \overline{AB}$? Why is $\overline{OC} \perp \overline{CB}$?

   Answer: $\overline{OA} \perp \overline{AB}$ and $\overline{OC} \perp \overline{CB}$ because radii and tangents are perpendicular and form 90° angles.

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c. Select ^ to show the next steps. Why is $\Delta AOB \cong \Delta COB$?

Answer: $\overline{OB}$ is congruent to itself, $\overline{OA} \cong \overline{OC}$. Therefore, $\Delta AOB \cong \Delta COB$ by the Hypotenuse-Leg Theorem.

> **Teacher Tip:** Make sure students go through all of the steps including the last step. In the last step, they will need to drag the point (point D) that transforms ΔAOB to ΔCOB. Discuss which transformations are being used.

d. Why can you conclude $\overline{AB} \cong \overline{CB}$?

Answer: Corresponding parts of congruent triangles are congruent.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The difference between a secant line and a tangent line.
- Tangent lines are perpendicular to a radius at the point of tangency.
- When tangent lines intersect at a point outside the circle, the tangent segments are congruent.
- The proof used to prove tangent segments congruent.

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Note 1

Question 3a, *Quick Poll:* Send students a *True/False Quick Poll:*
When *B* is outside the circle on the diagram on page 3.1, $\overleftrightarrow{AB} \cong \overleftrightarrow{CB}$.

Answer: False. $\overline{AB} \cong \overline{CB}$. Lines are not congruent.

Note 2

Question 4, *Live Presenter:* You might make a student the *Live Presenter* to click through each of the steps in question 4 so that you can go through the proof as a whole class.

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