

# Consul Educated Monkey



## Student Activity

7 8 9 **10** 11 12



TI-Nspire



Investigation



Student



40 min

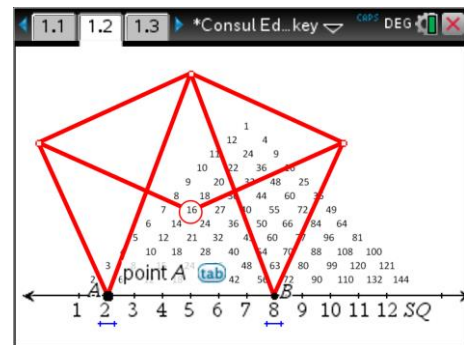
## Investigation

Consul the Educated Monkey is a mechanical toy that was invented in 1916 to help students learn their multiplication tables. Students move the monkey's feet to select two integer values and the monkey's hands point to the answer. The geometrically designed toy can be used for more than multiplication. In this investigation you will determine how and why the device works and design further applications.

Open the TI-Nspire file: Consul Educated Monkey

Navigate to Page 1.2. Use the navigation pad to move the mouse over either point A or Point B corresponding to Consul's feet. When the mouse changes to an open-hand, click and hold (or press Ctrl and Click) to grab the monkey's feet. Move the feet left / right to check the accuracy of the model and observe how the linkages move.

**Note:** In the physical version Consul's legs cannot touch or cross.



### Question: 1.

Place Consul's left leg on 3. Move Consul's right leg from 4 through to 12. What do you notice about the movement of the circle that highlights the answer to each multiplication problem?

### Question: 2.

Place Consul's right leg on a number greater than 8, then move Consul's left leg. What do you notice about the movement of the circle that highlights the answer?

### Question: 3.

Can the entire multiplication tables  $1 \times 1$  through to  $12 \times 12$  be generated using Consul?

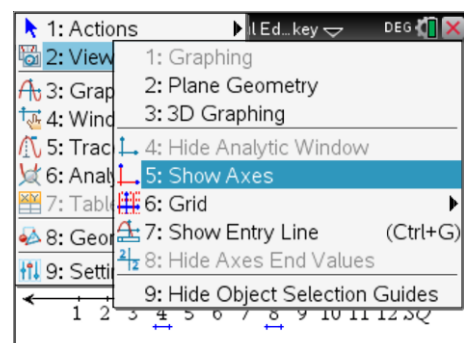
## How does it work – Part 1

Navigate to page 2.1.

The Consul construction is available minus the highlight circle and background numbers. A set of Axis can be used to track the location of point P (Consul's hands).

Move point A to (1, 0).

Grab point B and move it along the x axis. Write down the coordinates of point P for each location of point B.



**Question: 4.**

Record the location of point B and corresponding location of point P in the table below. Point A = (1, 0)  
 In the table below  $P_x$  = x coordinate of point P and  $P_y$  = y coordinate of point P. Determine an equation relating  $(P_x, P_y)$  and graph your equation. Once again, drag point B to ensure the equation is correct.

<b>Point B</b>	2	3	4	5	6	7	8	9	10	11	12
<b><math>P_x</math></b>	1.5										
<b><math>P_y</math></b>	0.5										

**Question: 5.**

Set Point A at (2, 0). Determine the equation for the new line passing through the set of points:  $(P_x, P_y)$ .  
 Graph your equation and drag point B to ensure the equation is correct.

<b>Point B</b>	3	4	5	6	7	8	9	10	11	12
<b><math>P_x</math></b>										
<b><math>P_y</math></b>										

**Question: 6.**

Suppose point A is at (5, 0), without constructing a table of values, what do you think the equation might be relating point  $(P_x, P_y)$ . Check your answer by graphing it and moving point B accordingly.

**Question: 7.**

Explore how point P moves when point B is located at (10, 0) and point A is moved. Determine the corresponding motion point P.

**How does it work – Part 2**

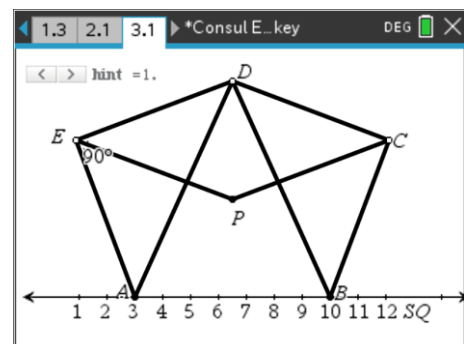
So far it would appear that point P moves in a straight line. We can prove this by knowing how the linkages in Consul are constructed and combine a little geometric reasoning.

Here are a few things about the physical construction:

$\angle AED = \angle BCD = 90^\circ$

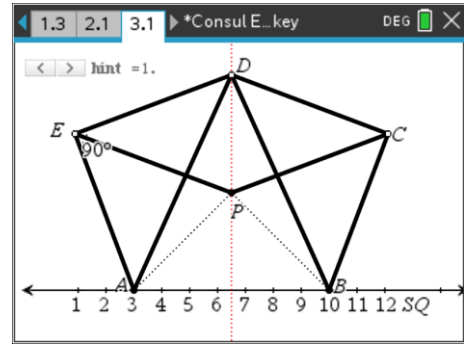
Distance:  $AE = EP = ED = BC = CP = CD$

Consul is symmetrical about the line passing through points D and P.



In this first stage it will be useful to prove that  $\angle APB = 90^\circ$ .

The questions below provide a guide as to how to proceed with such a proof. The TI-Nspire file can also be used to step through the hints.



**Question: 8.**

- a) What type of triangle is  $\triangle APE$ ?
- b) Write a relationship between  $\angle EAP$  and  $\angle APE$ .
- c) What type of triangle is  $\triangle EPD$ ?
- d) Write a relationship between  $\angle EPD$  and  $\angle PDE$ .
- e) Determine the value of  $\angle APE + \angle EPD$ . [Note  $\angle APE + \angle EPD = \angle APD$ ]
- f) Given Consul is symmetrical, determine the value of  $\angle APB$ .

**Question: 9.**

Given the symmetry of Consul and  $\angle APB = 90^\circ$ , determine  $\angle BAP$ .

**Question: 10.**

Let point Q be the midpoint of AB. If length AQ is doubled, what happens to length QP? (Explain)

**Question: 11.**

Suppose Consul is superimposed onto the Cartesian plane (as per Page 2.1 of the TI-nspire document), what does your answer to Question 10 say about the calculation:  $\frac{\Delta y}{\Delta x}$  and hence the movement of point P.

**Question: 12.**

Use your answers to help explain how Consul works and show how Consul could be used for basic addition.